



Chapter 8

Predicting What Comes Next: Exploring Sequences and Progressions

Complete Notes, Formulas (Ganita Manjari, Grade 9, Part I)

Part A: Chapter Notes

8.1 Introduction to Sequences

Sequences are patterns formed by numbers (or other objects) arranged in a particular order. Understanding sequences helps us predict how numbers grow, shrink, or repeat.

Some familiar sequences are:

- 1, 2, 3, 4, 5, 6, ... — Natural Numbers
- 1, 3, 5, 7, 9, 11, ... — Odd Numbers
- 1, 3, 6, 10, 15, 21, ... — Triangular Numbers
- 1, 4, 9, 16, 25, 36, ... — Square Numbers

A sequence is an ordered list of numbers, where each number is called a term. A sequence may be finite (e.g. 6, 12, 24, 48, 96) or infinite (continues indefinitely, shown using three dots ...).

Triangular and Square Numbers

Each term of the triangular number sequence is the sum of the natural numbers up to that term: $1 = 1$, $3 = 1+2$, $6 = 1+2+3$, $10 = 1+2+3+4$, $15 = 1+2+3+4+5$, and so on.

The First Five Triangular Numbers (1, 3, 6, 10, 15)



Fig. 8.1: The first five triangular numbers, shown as triangular dot arrays



Each term of the square number sequence is the sum of the odd numbers up to that term: $1 = 1$, $4 = 1+3$, $9 = 1+3+5$, $16 = 1+3+5+7$, and so on.

Notation for Sequences

We use $t_1, t_2, t_3, \dots$ to denote the 1st, 2nd, 3rd, ... terms of a sequence (subscripts match term position). Different sequences may use different letters, e.g. $s_1, s_2, \dots$ or $u_1, u_2, \dots$. The position n is always a non-negative integer, but the term t_n itself can be any real number (positive, negative, or a fraction).

8.2 Explicit Rule for a Sequence

An explicit formula uses the position number n directly to calculate the value of the n th term.

Example: $u_n = 2n - 1$ is the explicit rule for the odd number sequence ($u_1=1$, $u_2=3$, $u_3=5$, ...)

Using an explicit formula, we can find any term directly

8.3 Recursive Rule for a Sequence

A recursive formula (or recursive rule) describes a sequence by relating each term to the previous term(s). To find a term, the earlier term(s) must already be known.

General recursive form: $t_1 = (\text{first value given})$, $t_n = (\text{rule involving } t_{n-1}, t_{n-2}, \dots)$ for $n \geq 2$

Example: $t_1 = 1$, $t_n = t_{n-1} + 3$ for $n \geq 2$, generates 1, 4, 7, 10, 13, ...

Virahāṅka–Fibonacci Sequence

A recursive rule can also involve the previous two (or more) terms. The most famous example:

$$V_1 = 1,$$

$$V_2 = 2,$$

$V_n = V_{n-1} + V_{n-2}$ for $n \geq 3$, giving the sequence 1, 2, 3, 5, 8, 13, 21, 34, ... — each term is the sum of the previous two. This was first studied by the Indian scholar Virahāṅka (7th century CE) in the context of Prakrit poetic meter, later by Gopāla and Hemachandra, and much later by the Italian mathematician Fibonacci (c. 1200 CE).

8.4 Arithmetic Progressions (AP)



An arithmetic progression is a sequence in which the difference between any two consecutive terms is constant. This constant is called the common difference (d).

General form of an AP: $a, a+d, a+2d, a+3d, \dots, a+(n-1)d$, where a = first term, d = common difference

Explicit (n th term) formula: $t_n = a + (n - 1) \times d$

Recursive formula: $t_1 = a, t_n = t_{n-1} + d$ for $n \geq 2$

If d is negative, the sequence is decreasing (e.g. 11, 7, 3, -1, -5, ... has $a = 11$, $d = -4$).

8.4.1 Visualising an AP

The growing pattern of squares below (Stages 1, 2, 3, 4) gives the sequence 1, 5, 9, 13, ... — an AP with $a = 1$ and $d = 4$, so $t_n = 4n - 3$.

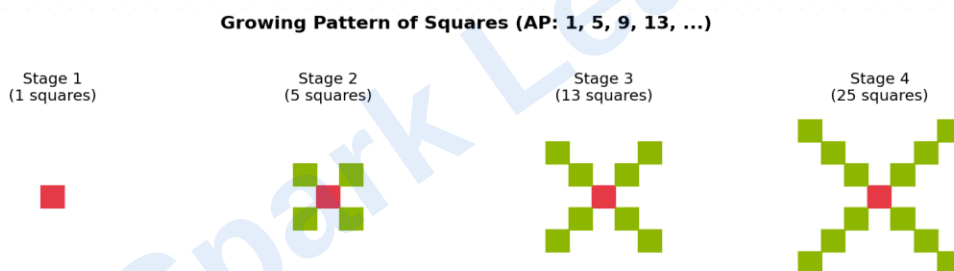


Fig. 8.3: A growing pattern of squares representing the AP 1, 5, 9, 13, ...

When the ordered pairs (stage number, number of squares) are plotted, they lie exactly on a straight line. This is a key visual feature of every AP — its graph is always linear.

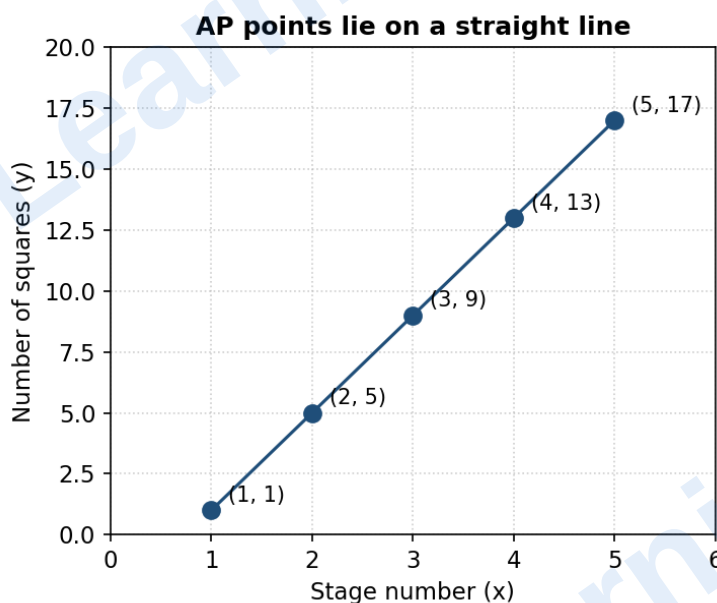


Fig. 8.4: Points from an AP always lie on a straight line

8.5 Sum of the First n Natural Numbers

The sum of n natural numbers :1, 2, 3, ..., n is given by : (a technique attributed to Āryabhaṭa, Chapter 2, Verse 19 of the Āryabhaṭīya),

$$S_n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

This formula is also the explicit rule for the triangular number sequence: $t_n = \frac{n(n+1)}{2}$.

It can also be used to find the sum of consecutive numbers in any range, e.g. $25 + 26 + \dots + 58 = S_{58} - S_{24}$.

8.6 Geometric Progressions (GP)

A geometric progression is a sequence in which each term after the first is obtained by multiplying the previous term by a fixed non-zero number, called the common ratio (r). The ratio of any two consecutive terms is constant.

General form of a GP: $a, ar, ar^2, ar^3, \dots, ar^{n-1}$, where a = first term, r = common ratio

Explicit (nth term) formula: $t_n = a \times r^{n-1}$

Recursive formula: $t_1 = a, t_n = r \times t_{n-1}$ for $n \geq 2$



8.6.1 Fun with Fractals — the Sierpiński Triangle

Starting from an equilateral triangle (Stage 0), we repeatedly remove the central triangle formed by joining the midpoints of the sides. This produces a fractal pattern called the Sierpiński triangle.

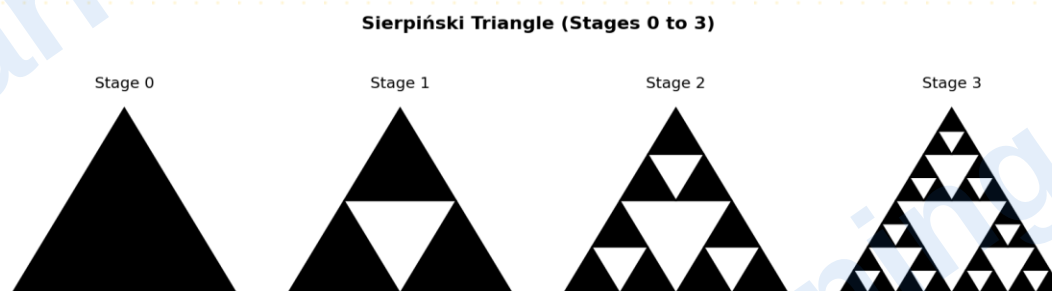


Fig. 8.7: The Sierpiński triangle, Stages 0 to 3

Number of black triangles at Stage n : $t_n = 3^n$ (a GP with $a = 1$, $r = 3$) — grows rapidly.

Area of the black region at Stage n (if Stage 0 has area 1 sq. unit): $s_n = (3/4)^n$ (a GP with $a = 1$, $r = 3/4$) — shrinks towards 0 as n increases.

8.6.2 Visualising a GP

The growing pattern of squares below gives the sequence 3, 6, 12, 24, ... — a GP with $a = 3$, $r = 2$, so $t_n = 3 \times 2^{n-1}$.

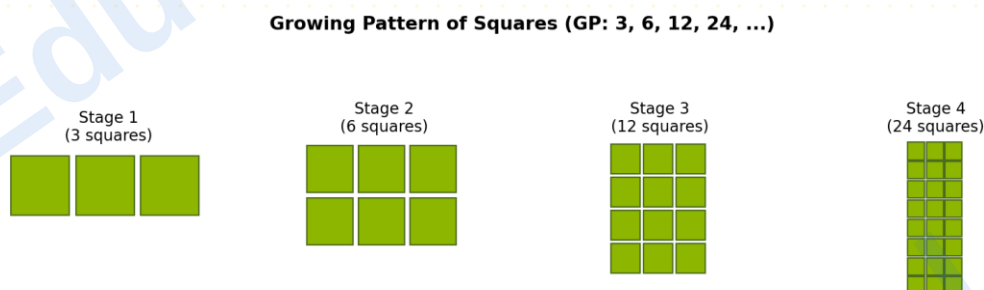


Fig. 8.6: A growing pattern of squares representing the GP 3, 6, 12, 24, ...

Unlike an AP, when the ordered pairs (stage number, number of squares) of a GP are plotted, they do NOT lie on a straight line — they lie on a curve that rises increasingly steeply (exponential growth).

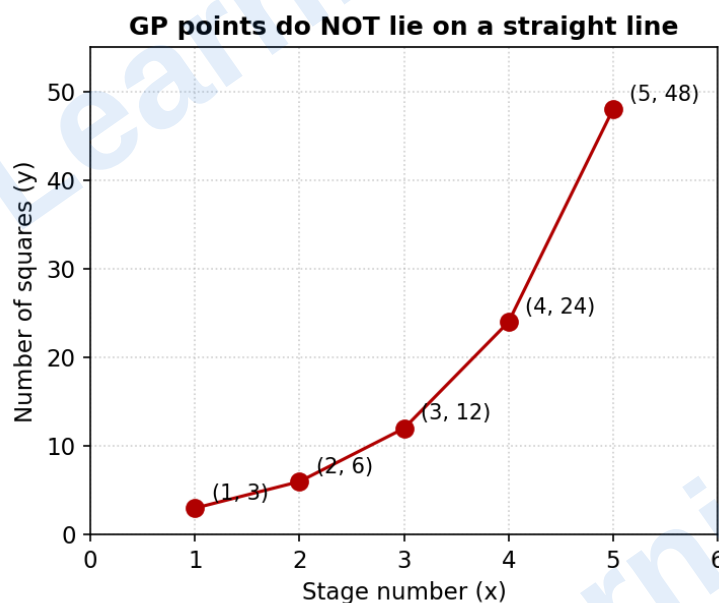


Fig. 8.9: Points from a GP do NOT lie on a straight line

Chapter Summary — Key Formulas

Explicit formula: rule using position n directly, e.g. $t_n = 2n - 1$

Recursive formula: rule using previous term(s), e.g. $t_1 = a$, $t_n = t_{n-1} + d$

Triangular numbers: $t_n = n(n + 1) / 2$

AP — n th term: $t_n = a + (n - 1)d$ | General form: $a, a+d, a+2d, \dots$

Sum of first n natural numbers: $S_n = n(n + 1) / 2$

GP — n th term: $t_n = a \times r^{n-1}$ | General form: $a, ar, ar^2, \dots$