

Class 10 Maths

Chapter 2: Polynomials

1. Introduction and Degree of a Polynomial

An algebraic expression in which the variables have only non-negative integer powers is called a polynomial.

- Degree of a Polynomial: The highest power of the variable x in the polynomial $p(x)$ is called the degree of the polynomial.

- Linear Polynomial: A polynomial of degree 1 (e.g., $ax + b$).
- Quadratic Polynomial: A polynomial of degree 2 (e.g., $ax^2 + bx + c$).
- Cubic Polynomial: A polynomial of degree 3 (e.g., $ax^3 + bx^2 + cx + d$).

2. Geometrical Meaning of the Zeroes of a Polynomial

- Zero of a Polynomial: A real number " k " is said to be a zero of a polynomial $p(x)$ if $p(k) = 0$.

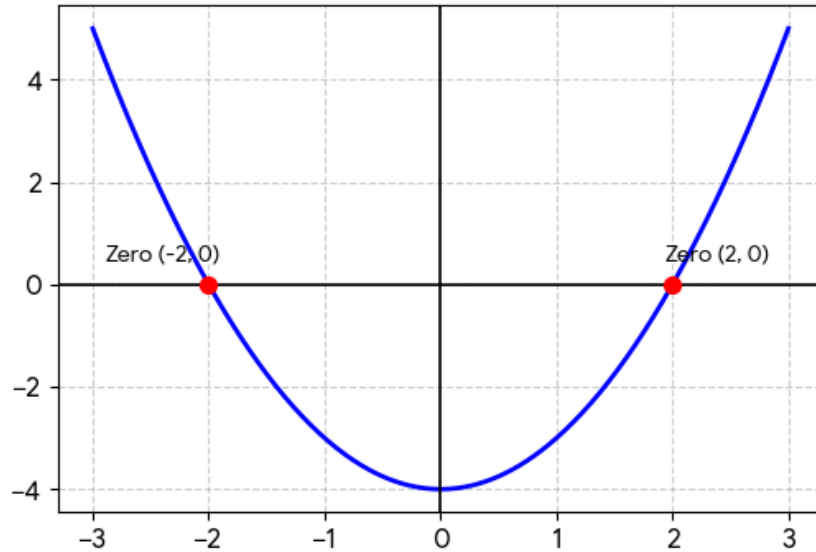
- Geometrically, the zeroes of a polynomial $p(x)$ are precisely the x -coordinates of the points where the graph of $y = p(x)$ intersects the X -axis.

- Number of Zeroes: A polynomial of degree " n " can intersect the x -axis at most " n " times. Therefore, it can have at most " n " real zeroes.

- A linear polynomial has exactly 1 zero. (Line will intersect the x -axis at one point only)

- A quadratic polynomial can have at most 2 zeroes.

Geometrical Meaning of Zeroes (Quadratic Polynomial)



In the graph above, the parabola $y = x^2 - 4$ intersects the X-axis at two points: $x = -2$ and $x = 2$. Therefore, -2 and 2 are the zeroes of this quadratic polynomial.

3. Relationship Between Zeroes and Coefficients

For a Quadratic Polynomial: $p(x) = ax^2 + bx + c$ (where $a \neq 0$)

Let α and β be the two zeroes of the quadratic polynomial. Then the relationship between its zeroes and coefficients is given by:

- Sum of Zeroes $(\alpha + \beta) = -b / a = -(\text{Coefficient of } x) / (\text{Coefficient of } x^2)$
- Product of Zeroes $(\alpha \times \beta) = c / a = (\text{Constant term}) / (\text{Coefficient of } x^2)$

4. Formation of a Quadratic Polynomial

If the zeroes (α and β) or their sum and product are given, the quadratic polynomial can be formed using the formula:

$$p(x) = k [x^2 - (\text{Sum of zeroes})x + (\text{Product of zeroes})]$$

$$p(x) = k [x^2 - (\alpha + \beta)x + \alpha\beta]$$

Where "k" is any non-zero real constant.