

Class 10 Maths

Chapter 1: Real Numbers

1. Introduction to Real Numbers

Real numbers constitute the union of all rational and irrational numbers.

- Rational Numbers: Numbers that can be expressed in the form p/q , where p and q are integers and $q \neq 0$. (e.g., 2, $-3/5$, 0)
- Irrational Numbers: Numbers that cannot be expressed in the form p/q . (e.g., $\sqrt{2}$, $\sqrt{3}$, π)

2. The Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Example: Prime factorisation of $210 = 2 \times 3 \times 5 \times 7$

3. HCF and LCM (Prime Factorisation Method)

- HCF (Highest Common Factor): Product of the smallest power of each common prime factor in the numbers.
- LCM (Lowest Common Multiple): Product of the greatest power of each prime factor, involved in the numbers.

Relationship between HCF and LCM:

For any two positive integers a and b :

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

(Note: This formula is only valid for two numbers, not for three.)

4. Revisiting Irrational Numbers (Proof by Contradiction)

Theorem: Let p be a prime number. If p divides a^2 , then p divides a , where a is a positive integer.

Key steps to prove $\sqrt{2}$ is irrational:

1. Assume to the contrary that $\sqrt{2}$ is rational.
2. Then, $\sqrt{2} = a/b$ (where a and b are co-prime integers and $b \neq 0$).
3. Squaring both sides: $2 = a^2/b^2 \Rightarrow a^2 = 2b^2$
4. This means 2 divides a^2 , so 2 also divides a .
5. Let $a = 2c$. Substituting this gives $(2c)^2 = 2b^2 \Rightarrow 4c^2 = 2b^2 \Rightarrow b^2 = 2c^2$
6. This means 2 divides b^2 , so 2 also divides b .
7. Since 2 divides both a and b , they are not co-prime (contradiction).
8. Therefore, our assumption was wrong, and $\sqrt{2}$ is an irrational number.

5. Key Takeaways

- The HCF of two co-prime numbers is always 1.
- The sum or difference of a rational and an irrational number is always irrational. (e.g., $2 + \sqrt{3}$ is irrational).
- The product or quotient of a non-zero rational and an irrational number is always irrational. (e.g., $3\sqrt{2}$ is irrational).