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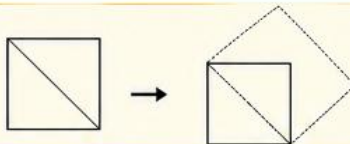
THE BAUDHĀYANA-PYTHAGORAS THEOREM

In this chapter

- Doubling a Square
- Halving a Square
- Hypotenuse of an Isosceles Right Triangle
- Value of $\sqrt{2}$
- Pythagoras Theorem

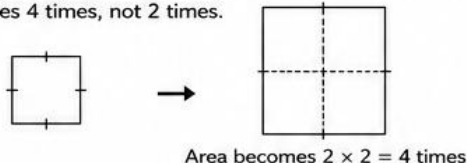
Key Idea (Baudhāyana's Insight)

The diagonal of a square produces a square of double the area of the original square.



1. DOUBLING A SQUARE

- If each side of a square is doubled, the area becomes 4 times, not 2 times.



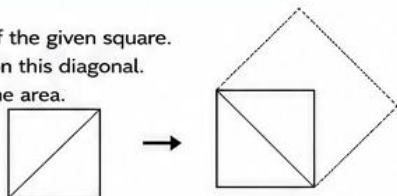
Area becomes $2 \times 2 = 4$ times

Baudhāyana's Theorem (Śulba-Sūtra, 1.9)

The diagonal of a square produces a square of double the area of the original square.

Construction

Draw the diagonal of the given square. Construct a square on this diagonal. It will have double the area.



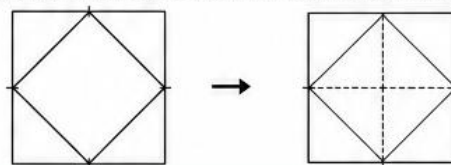
Why does the new square have double the area?

- Draw horizontal and vertical lines through the midpoints of the sides.
- These lines pass through the vertices of the dotted square because the sides of the original square bisect the angles of the new square.
- The original square is made of 2 congruent small triangles; the new square is made of 4 congruent small triangles.
- Hence, $\text{Area}(\text{new square}) = 2 \times \text{Area}(\text{original square})$.

2. HALVING A SQUARE

Construction

Draw a square inside the given square by joining the midpoints of the sides. The inner square has half the area.

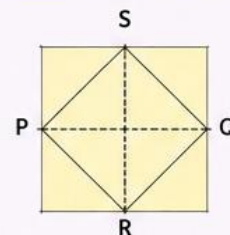


Why does the inner square have half the area?

- The diagonals of the inner square divide the figures into 4 congruent right isosceles triangles.
- The outer square is made of 4 such triangles.
- The inner square is made of 2 such triangles.
- Hence, $\text{Area}(\text{inner square}) = \frac{1}{2} \times \text{Area}(\text{outer square})$.

Halving a Square Using Paper

1. Take a square sheet of paper.
2. Fold it inward so that the crease lines pass through the midpoints of the sides.
3. The inner square PQRS is the required square with half the area.



3. HYPOTENUSE OF AN ISOSCELES RIGHT TRIANGLE

Consider the isosceles right triangle PEAR.

Legs = 1 unit each.

Let $ER = c$ be the hypotenuse.

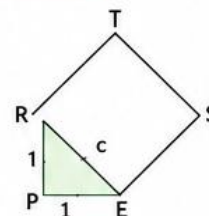
- A square of side 1 is made of two such triangles.
- The square constructed on the diagonal (hypotenuse) has twice the area of the original square.

So, $\text{Area of square REST} = 2 \times \text{Area of square PEAR}$
 $= 2 \times 1 = 2$ sq. units.

But the area of square $REST = c^2$.

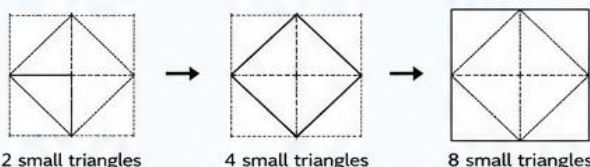
Therefore, $c^2 = 2 \Rightarrow c = \sqrt{2}$.

Hence, the hypotenuse of an isosceles right triangle with unit legs is $\sqrt{2}$ units.



Sequence of Squares

Each square has double the area of the previous one.



2 small triangles

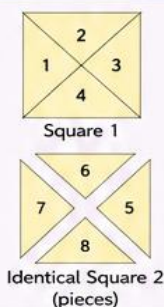
4 small triangles

8 small triangles

Areas in ratio: $1 : 2 : 4 : 8 : \dots$

Doubling a Square Using Paper

1. Cut two identical squares of paper.
2. In Square 1, draw both diagonals and cut into 4 pieces (1, 2, 3, 4).
3. In identical Square 2, cut along the diagonals to get pieces (5, 6, 7, 8).
4. Place pieces 5, 6, 7, and 8 around Square 1. The outer shape is a square with double the area.



Value of $\sqrt{2}$ (Decimal Representation)

We know: $1^2 = 1$, $2^2 = 4 \Rightarrow 1 < \sqrt{2} < 2$.

Using the method of closer bounds:

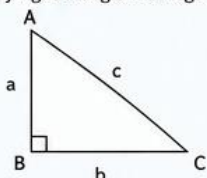
Lower bound (a)		Upper bound (b)	
a	a^2	b	b^2
1.4	1.96	1.5	2.25
1.41	1.9881	1.42	2.0164
1.414	1.999396	1.415	2.002225
1.4142	1.9996164	1.4143	2.0002449

Thus, $\sqrt{2} \approx 1.4142$ (to 4 decimal places).

It is an irrational number (non-terminating, non-repeating decimal).

4. PYTHAGORAS THEOREM (THE FINAL THEOREM)

In any right triangle with legs a , b and hypotenuse c (opposite the right angle):



The square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2$$

Key Definitions

- **Square:** A quadrilateral with all sides equal and all angles 90° .
- **Diagonal of a square:** A line segment joining two opposite vertices.
- **Hypotenuse:** The side opposite to the right angle in a right triangle.
- **East-west line:** A horizontal line.
- **North-south line:** A vertical line.
- **Congruent figures:** Figures that have the same shape and size.

Important Takeaways

- ✓ Diagonal of a square $\rightarrow$ new square with double the area.
- ✓ Connecting midpoints $\rightarrow$ new square with half the area.
- ✓ Hypotenuse of an isosceles right triangle with unit legs $= \sqrt{2}$.
- ✓ $1 < \sqrt{2} < 2$ and $\sqrt{2} \approx 1.4142$.
- ✓ In any right triangle, $c^2 = a^2 + b^2$.

THE BAUDHAYANA – PYTHAGORAS THEOREM

WORKSHEET

SECTION A: PART 1 MCQS (10 × 1 = 10 MARKS)

1. In a triangle with sides 3.10 cm, 3.01 cm, and 3.001 cm, which is the longest side?
(a) 3.10 cm (b) 3.01 cm (c) 3.001 cm (d) All are equal
2. A triangle having one angle equal to 90° is called a:
(a) Acute triangle (b) Obtuse triangle (c) Right-angled triangle (d) Equilateral triangle
3. The sum of the interior angles of a triangle is:
(a) 90° (b) 180° (c) 270° (d) 360°
4. What is the value of $8^2 + 8^2$ is
(a) 16^2 (b) 8^4 (c) 64 (d) 128
5. What is the square root of 49?
(a) 6 (b) 7 (c) 8 (d) 9
6. Which of the following is a perfect square?
(a) 20 (b) 25 (c) 30 (d) 35
7. The diagonal of a rectangle forms:
(a) two equilateral triangles (b) two isosceles triangles
(c) two right-angled triangles (d) two obtuse triangles
8. What is the value of $13^2 - 5^2$ is
(a) 225 (b) 144 (c) 169 (d) 81
9. In a right-angled triangle, the longest side will be :
(a) Base (b) Height (c) Hypotenuse (d) Median
10. Which of the following is NOT a Perfect Square?
(a) 144 (b) 164 (c) 324 (d) 484

PART 2 MCQS (10 × 1 = 10 MARKS)

11. There can be _____ Baudhayan Triples
(a) 1 (b) 10 (c) finite (d) infinitely many
12. Which of the following sets of numbers forms a Baudhayan-Pythagorean triple?
(a) (8,24,25) (b) (5,12,13) (c) (6,8,11) (d) (3,4,6)
13. According to the Baudhayan Sulba Sutra, if a diagonal of a rectangle is c and its sides are a and b , the relationship between them is:
(a) $a+b=c$ (b) $a^2 - b^2 = c^2$ (c) $c^2 - b^2 = a^2$ (d) $a \times b = c^2$
14. The value of $\sqrt{2}$ lies between:
(a) 0 and 1 (b) 1 and 2 (c) 1.4 and 1.5 (d) 1.41 and 1.42
15. There may be "two right angles in a triangle or two obtuse angles in a triangle" this statement is ---
(a) Always true (b) Always false (c) Sometimes true (d) sometimes false
16. If the sides of a triangle are in the ratio 3:4:5, the triangle must be:
(a) Acute-angled (b) Obtuse-angled (c) Right-angled (d) Equilateral
17. A ladder 10 m long reaches a window of a building 8 m above the ground. The distance of the foot of the ladder from the base of the building is:
(a) 6 m (b) 7 m (c) 8 m (d) 9 m
18. State to true or false – "The Baudhayan Sulba Sutra holds for all triangles."
(a) Always TRUE (b) Always FALSE (c) Sometimes TRUE (d) can't say
19. Who finally proved Fermat's last theorem?
(a) Euclid (b) Pythagoras (c) Bodhayana (d) Andrew Wiles
20. In the right angled triangle, angle opposite to right angle is called
(a) Attitude (b) Median (c) Hypotenuse (d) Base

WORKSHEET-2

Section B: Short Answer ($10 \times 2 = 20$ marks)

1. A man goes 15 m due east and then 20 m due north. How far is he from his starting point ?

2. If $(k, 12, 13)$ is a Baudhayana-Pythagorean triple, find the value of k .

3. True or False: "Any scalar multiple of a Pythagorean triplet (e.g., $2 \times (3, 4, 5)$) also forms a Pythagorean triplet. Write two such triplets.

4. In an isosceles right-angled triangle, if the hypotenuse is 82 cm, find the length of its equal sides.

5. If the sides of a triangle measure 8 cm, 15 cm, and 16 cm, determine whether it forms a right-angled triangle.

6. Calculate the height of an equilateral triangle whose side length is 6 cm.

7.Determine Which of these triplet is a Primitive Baudhayan triple.(a) (3,4,5) (b) (9,12,15)

8.If a side of an equilateral triangle is a, then what is its altitude?

9.If the diagonal of a rectangle is 13 cm and its width is 5 cm, what is its area?

10.Find the diagonal of a square is $20\sqrt{2}$ cm ,what is the side?

Section C: Short Answer ($6 \times 3 = 18$ marks)

11. A wire is attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

12. Find the length of the altitude of an isosceles triangle whose equal sides are 13 cm each and the base is 10 cm.

13. A 10 m long ladder is leaned against a vertical wall. If the base of the ladder is 6 m away from the wall, how high up the wall does the ladder reach?

14. An isosceles triangle can be split into two right-angled triangles. If the height of the triangle is 4 cm and the slant side (hypotenuse) is 5 cm:

(a) Find the half-base

(b) What is the total length of the full base?

15. A television screen size is measured diagonally. If a TV screen is 40 inches wide and 30 inches tall, what is the advertised size of the TV (diagonal length)?

16. A soccer field is a rectangle 90 meters wide and x meters long. The coach asks players to run from one corner to the other corner diagonally across and if players run a distance of 150 m. What is the length of field?

Section D: Long Answer ($4 \times 5 = 20$ marks)

17.State and prove the **Baudhayan-Pythagoras Theorem** . Is the hypotenuse of the right triangle longest side of it? Justify you answer.Give an example of Pythagoras triple which is non primitive.

18.Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

19.The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter using the Baudhayan theorem.

20. Write three more steps of following pattern. Also show 11^2 and 13^2 as sum of consecutive odd numbers as given in pattern

Perfect Squares and Odd Numbers

$$\begin{aligned}1 &= 1 = 1^2 \\1 + 3 &= 4 = 2^2 \\1 + 3 + 5 &= 9 = 3^2 \\1 + 3 + 5 + 7 &= 16 = 4^2 \\1 + 3 + 5 + 7 + 9 &= 25 = 5^2 \\1 + 3 + 5 + 7 + 9 + 11 &= 36 = 6^2\end{aligned}$$

Section E: Case Study-Based Questions (2×4=8)

21. Construction of Vedic Fire Altars (Chiti)

In ancient India, the Sulba Sutras laid out instructions for building ritualistic **fire altars called Chiti**. Baudhayan documented precise structural specifications to build a square altar that possessed an area identical to the combined total area of two smaller individual square altars.



- Altar 1 has a side length of 6 cubits (units).
- Altar 2 has a side length of 8 cubits (units).

Questions: (i) What is the sum of the areas of Altar 1 and Altar 2?

- (ii) Using Baudhayan's rule, what should be the side length of the new combined square fire altar?
- (iii) What geometric dimension of the two original squares corresponds directly to the side length of the newly constructed square altar?

22. The Broken Tree (Natural Disaster)

During a thunderstorm, a tall eucalyptus tree snaps due to heavy wind forces. The tree does not break completely off its base; instead, it bends over.

- The broken top part of the tree touches the flat ground at a distance of 9 meters from the base of the trunk.
- The point where the trunk snapped and bent is at a height of 12 meters from the ground.

Questions: (i) Identify the right-angled relationship established by this real-world scenario.

(ii) Find the total length of the broken part of the tree that is leaning toward the ground.

(iii) Determine the original, full height of the tree before it sustained damage.

Self Assessment:

I understood all the concepts:

☐ Yes

☐ No

I need more practice:

☐ Yes

☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood all the concepts:

☐ Yes

☐ No

He/She need more practice:

☐ Yes

☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers of Worksheet-1

Q	1	2	3	4	5	6	7	8	9	10
Ans	a	c	b	d	b	b	c	b	c	b
Q	11	12	13	14	15	16	17	18	19	20
Ans	d	B	c	d	b	c	a	B	d	c

Answers of Worksheet-2

Q	1	2	3	4	5
Ans	25 m	K = 5	True (6,8,10) (12,16,20)	$41\sqrt{2}$	no
Q	6	7	8	9	10
Ans	$3\sqrt{3}$ cm	(a)	$(\sqrt{3}/2)$	Side = 12cm area = $12 \times 5 = 60$	20 cm
Q	11	12	13	14	15
Ans	$6\sqrt{7}$ cm	12	8 m	3, 6	50 inch
Q	16	17	18	19	20
Ans	120 m	---	13 m	68	-----
Q	21			22	
Ans	The diagonal of a rectangle formed by the sides of the two original squares corresponds directly to the side length of the newly constructed square altar			15 m	