

Class 10 Maths

Chapter 3: Pair of Linear Equations in Two Variables

1. Introduction & General Form

An equation which can be put in the form $ax + by + c = 0$, where a , b and c are real numbers, and a and b are not both zero, is called a linear equation in two variables x and y .

The general form for a pair of linear equations in two variables x and y is:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

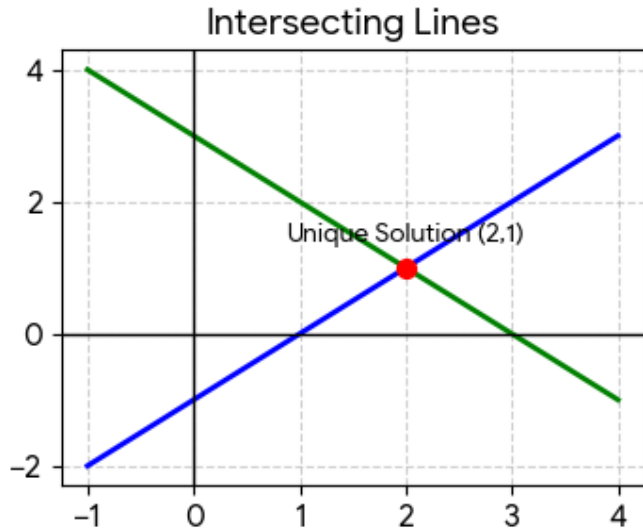
Where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers and $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$.

2. Graphical Representation and Solutions

The graph of a pair of linear equations in two variables is represented by two lines. There are three possibilities:

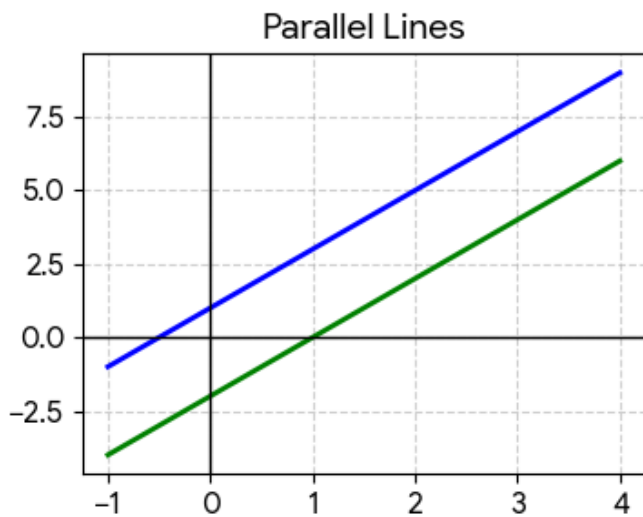
Case 1: Intersecting Lines

- Algebraic Condition: $a_1/a_2 \neq b_1/b_2$
- Solution: The system has exactly one (unique) solution where the lines intersect.



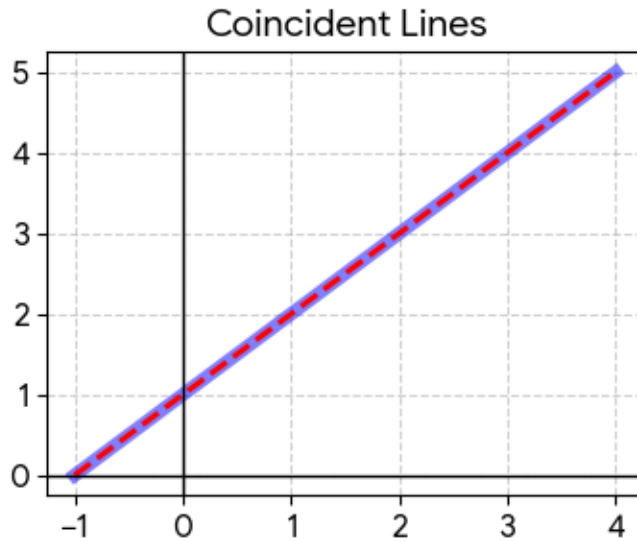
Case 2: Parallel Lines

- Algebraic Condition: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$
- Solution: The system has NO solution (inconsistent). The lines never meet.



Case 3: Coincident Lines

- Algebraic Condition: $a_1/a_2 = b_1/b_2 = c_1/c_2$
- Solution: The system has infinitely many solutions (dependent/consistent). The lines overlap exactly.



3. Algebraic Conditions for Solutions

Comparing the ratios of coefficients (a_1/a_2 , b_1/b_2 , c_1/c_2):

1. Intersecting Lines (Unique Solution): $a_1/a_2 \neq b_1/b_2$
2. Coincident Lines (Infinitely Many Solutions): $a_1/a_2 = b_1/b_2 = c_1/c_2$
3. Parallel Lines (No Solution): $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

4. Algebraic Methods of Solving

When a graphical solution is not convenient, we use algebraic methods:

- Substitution Method: Express one variable in terms of the other from one equation, and substitute this expression in the second equation to find the value of the variables.
- Elimination Method: Multiply one or both equations by suitable non-zero constants to make the coefficients of one variable numerically equal. Then add or subtract one equation from the other to eliminate that variable.