KENDRIYA VIDYALAYA SANGATHAN

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

CBA TEST ITEMS

MATHEMATICS

CLASS 10

ZONAL INSTITUTE OF EDUCATION AND TRAINING
MYSURU

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DIRECTOR'S MESSAGE……

It is with profound delight and utmost pride that we present the Competency Based Assessment question bank for **CLASS 10** which was prepared by TGT(Mathematics) of the feeder regions during the 03 – day workshop on "**Competency Based Assessment in Mathematics: Design of test items**" It's my firm belief that access to quality education should know no boundaries, transcending social and economic constraints. Our collective vision is to empower all students and teachers with the tools for success and intellectual growth.

With their steadfast dedication, the TGT(Mathematics) from the feeder Regions namely Bangalore, Chennai, Ernakulam and Hyderabad have invested their knowledge and expertise in preparation of the CBA test items.

It is with pleasure that I place on record my commendation for the commitment and dedication of the team of TGT(Mathematics) from the four Regions, Shri. Siby Sebastian, Principal KV INS Dronacharya, Kochi, Ernakulam Region & Associate Course Director, the Resource persons Mr. M. S. Kumar Swamy, TGT(Maths), KV Gachibowli, Hyderabad & Ms P S Kavitha, TGT(Maths), K V DRDO Bengaluru and Mr. D. Sreenivasulu, Training Associate (Mathematics) from ZIET Mysore who has been the Coordinator of this assignment.

Wishing you all the very best in your academic journey!

MENAXI JAIN
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INDEX

S.NO	CHAPTER	PAGE NUMBERS
1	REAL NUMBERS	1 – 10
2	POLYNOMIALS	11 – 22
3	PAIR OF LINEAR EQUATIONS IN TWO VARIABLES	23 – 33
4	QUADRATIC EQUATIONS	34 – 44
5	ARITHMETIC PROGRESSIONS	45 – 51
6	TRIANGLES	52 – 63
7	COORDINATE GEOMETRY	64 – 73
8	INTRODUCTION TO TRIGONOMETRY	74 – 86
9	APPLICATION OF TRIGONOMETRY	87 – 104
10	CIRCLES	105 – 123
11	AREAS RELATED TO CIRCLES	124 – 134
12	SURFACE AREA AND VOLUMES	135 – 150
13	STATISTICS	151 – 175
14	PROBABILITY	176 - 183

Note: DESCRIPTION OF ASSESSMENT OBJECTIVE

AO1	Demonstrate knowledge and understanding of mathematical ideas, techniques and procedures
AO2	Apply knowledge and understanding of mathematical ideas, techniques and procedures to classroom and real-world situations

CHAPTER 1- REAL NUMBERS

MULTIPLE CHOICE QUESTIONS

1. The greatest possible length of the rope which can be used to measure two sticks of lengths 24 m and 18 m is (AO1)

a) 72 **b)** 12 **c)** 6 **d)** 3

2. If $x = 2^5 \times 7$, $Y = 2^2 \times 3^2 \times 5$ and $z = 3^n \times 5^2$ and **LCM** $(x,y,z) = 2^5 \times 3^4 \times 5^2 \times 7$, then **n =** (AO1)

a) 2 **b)** 3 **c)** 4 **d)** 5

3. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. The number of trees in each row will be (AO2)

a) 7 **b)** 8 **c**) 168 **d)** 42

4. Two positive integers 'a' and 'b' can be expressed as **a** = x^3y^2 and **b** = xy^3 ,x and y are prime numbers . What is the L.C.M of a and b? (AO2)

a) x^3y^2 **b)** xy^3 **c)** x^3y^3 **d)** xy^2

5. Meena has a 40 cm long red and 84 cm long yellow ribbon. She cuts each ribbon into pieces such that all pieces are of equal length. What is the length of each piece? (AO2)

a) 4 cm as it is the LCM of 40 and 84 **b)** 4 cm as it is the HCF of 40 and 84

c) 8 cm as it is the LCM of 40 and 84 **d)** 8 cm as it is the HCF of 40 and 84

6. **1245** is a factor of the numbers a and b. Which of the following will always have 1245 as a factor? (i) a + b (ii) a x b (iii) a ÷ b (AO2)

a) only (i) **b)** only (i) and (ii) **c)** only (iii) **d)** all (i),(ii) and (iii)

7. If p and q are two positive integers such that p – 10q = 0, then the HCF of p and q is. (AO2)

a) 0 **b)** p **c)** q **d)** pq

8. The LCM of two numbers is 1200.Which of the following cannot be their HCF? (AO1)

a) 600 **b)** 500 **c)** 400 **d)** 200

ASSERTION AND REASON QUESTIONS

(a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A).

(b) Both Assertion(A) and Reason(R) are true but the Reason(R) is **not** the correct explanation of Assertion(A).

(c) Assertion(A) is true, but Reason(R) is false.

(d) Assertion(A) is false, but Reason(R) is true

9. **Assertion:** 12^n ends with the digit zero, where n is any natural number.

Reason: Any number ends with digit zero, if its prime factor is of the form $2^m \times 5^n$, where m and n are natural numbers. (AO1)

10. Assertion: 3 x 5 x 11 x 7 + 5 is a composite number. (AO2)

Reason: A composite number has only two factors.

11. Assertion: 2 is a rational number (AO2)

Reason: The square root of all prime numbers are irrational.

12. Assertion: **(**2-√**5**) is an irrational number. (AO1)

Reason: The sum or difference of a rational and an irrational number is irrational.

2 MARKS QUESTIONS

13. 2 Bells toll together at 9.00 am. They toll after 8 and 12 minutes respectively. At what time will they toll together first time after 9:00 am? (AO2)

14. Show that 4^n can not end with the digit 0 for any natural number n. (AO1)

15. A grill has to be fixed to a window of length 60cm and breadth 36cm.what should be the maximum width between the grill bars so that spacing between the bars are equal? (AO2)

16. Given that $\sqrt{3}$ is irrational, prove that $5 - \sqrt{3}$ is irrational. (AO2)

17. The ratio of two numbers is 3: 4 and their H.C.F. is 4. Find their L.C.M. (AO2)

3 MARKS QUESTIONS

18. Prove that $\sqrt{3}$ is irrational. (AO1)

19. Ravi, Seema, and Raju start preparing cards for all the people in an old age home. They complete one card in 20, 16 and 10 minutes. If all of them started together at the same time, after what time will they start preparing a new card together? (AO2)

20. A worker needs to pack 350 kg of rice and 150 kg of wheat in bags such that each bag weighs the same. Each bag should either contain rice or wheat. find the greatest amount of rice/wheat the worker can pack in each bag? (AO2)

21. Two brands of chocolates are available in packs of 24 and 15 respectively. If I need to buy an equal number ofchocolates of both kinds, what is the least number of boxes of each kind I would need to buy? (AO2)

22. Three movie theaters are showing different movies. Theater A starts its movie every 80 minutes, Theater B every 120 minutes, and Theater C every 100 minutes. If all three movies start at the same time at 1:00 PM, at what time will they next start together? (AO2)

5 MARKS QUESTIONS

23. A school is organizing an event and wants to create gift bags containing pens, notebooks, and stickers. The school has 120 pens, 90 notebooks, and 150 stickers. What is the largest number of identical gift bags that can be prepared without any item left over, and how many of each item will be in each gift bag? (AO2)

24. Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. Find the sum of the three numbers. (AO2)

25. There are 24 peaches, 36 apricots and 60 bananas and they have to be arranged in several rows in such a way that every row contains the same number of fruits of only one type. (AO2)

(i)What is the minimum number of rows required for this to happen

(ii)How many trees would be there in each row?

CASE BASED QUESTIONS (4 MARKS EACH)

26. There are 104 students in Class X and 96 students in Class IX in a school. In the periodic test examination, students are to be evenly seated in parallel rows such that no two adjacent rows are of the same class. (AO2)

(i) Find the maximum number of parallel rows of each class for the seating arrangement.

(ii) Find the number of students of Class IX and also of Class X in a row.

27. Megha has 143 stamps; she gives away 11 stamps and divides the remaining equally into groups. Abdul has 220 stamps; he gives away 11 stamps and divides the remaining equally into groups. They end up with the same number of groups. (AO2)

(i) What is the number of groups?
(ii) What is the number of stamps in Megha and Abdul's groups?

28. Mrs. Latha teaches three dance classes. The first class has 24 students. The second class has 30 students. The third class has 18 students. Mrs. Latha wants to divide each class into groups so that every group in every class has the same number of students and there are no students left over. (AO2)

(i) What is the maximum number of students that she can put into each group ?

(ii) How many groups will second class have?

29. On Diwali, Megha decorated her balcony with colored bulbs. These bulbs are arranged in the pattern red, yellow and blue. The red bulb blinks after every 5 seconds, the yellow bulbs blink after every 8 seconds and the blue bulbs blink after every 10 seconds. (AO2)

(i)After how many seconds will all the bulbs blink together?

(ii) How many times will they blink together in 1 hour?

30. Vinod couldn't attend school for two days as he was not well. He found that his math's teacher had discussed ***Irrational numbers*** on those two days in the class. His friend briefed him about the topic.

'An **irrational number** is a non-terminating decimal that **cannot** be expressed as a fraction. This is in opposition to the definition of a rational number which is any number that **can** be expressed as a fraction in the form a/b where a and b are integers (whole numbers) and b ≠ 0. There are several famous irrational numbers. These include, (AO2)

π	Greek letter pi	3.141...	The ratio between the circumference and the diameter of a circle
φ	Greek letter phi	1.618...	The Golden Ratio
e	The number e	2.718...	Euler's Number
$\sqrt{2}$	The square root of 2	1.414...	A surd

Surds are one type of irrational numbers. A surd is the root of a number which produces a non-terminating decimal.'

For example,

√9=3 rational

√2=1.414… surd == irrational.

To check whether Vinod understood irrational numbers, his friend asked these questions.

(**i**).List out the irrational numbers from the list below

$$\sqrt{3},\ 6,\ \frac{8}{9},\ \pi,\ \frac{\sqrt{3}}{2},\ -0.4,\ e$$

(ii).The volume of Sphere is $V = 4/3\,\pi r^3$. Is V always, sometimes or never an irrational number? Give reasons for your answer

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SOLUTIONS: CHAPTER 1- REAL NUMBERS ANSWERS

MULTIPLE CHOICE QUESTIONS

1. C: 6cm
2. C: 4
3. A: 7
4. C: x^3y^3
5. B: 4 cm as it is the HCF of 40 and 84
6. B: only (i) and (ii)
7. C: q
8. B: 500

ASSERTION AND REASON QUESTIONS

9. D
10. C
11. D
12. A

2 MARKS QUESTIONS

13. LCM of 8 & 12 is 24, so the bell will toll together at 09:24 am.
14. A number is divisible by 10 if and only if it is divisible by both 2 and 5.
The prime factorisation of 4 does not have 5 , so it will not end with zero.
15. HCF of 60 and 36 is 12. the maximum width between the grill bars should be **12 cm**
16. Let us assume that 5 - √3 is a rational number. So, it can be written in the form a/b
5 -√3 = a/b , here a and b are coprime numbers and b≠0
Solving 5-√3 = a/b we get,
⇒√3 = 5 - a/b
RHS is a rational number. But we know that √3 is an irrational number.
So, it contradicts our assumption that 5-√3 is a rational number.
Therefore 5-√3 is an irrational number
17. Let the two numbers be 3x and 4x, where x is a common multiple.
Since their HCF is 4, we can say that x = 4. Now, the two numbers are 3x = 3 * 4 = 12 and 4x = 4 * 4 = 16.
LCM = (Product of the numbers) / (HCF of the numbers).
So, LCM = (12 * 16) / 4 = 192 / 4 = 48.
Therefore, the LCM of the two numbers is 48.

3 MARKS QUESTIONS

18. Let us assume that √2 is irrational number.
Let √2 = a / b where a, b are coprimes & b ≠ 0
Now √2 = a / b
⇒ $2 = a^2 / b^2$
⇒ $2b^2 = a^2$
⇒ a^2 is divisible by 2

$\Rightarrow$ a is divisible by 2
$\therefore$ let a = 2c
$a^2 = 4c^2 \Rightarrow 2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$
$\therefore b^2$ is divisible by 2
$\therefore$ b is divisible by 2
$\therefore$ a are b are divisible by 2.
this contradicts our supposition that a/b are coprimes
Hence our assumption is wrong
$\therefore \sqrt{2}$ is irrational number.

19. we need to find the **Least Common Multiple (LCM)**
LCM of 20,16 and 10 is 80

20. We need to find the HCF .
The HCF of 350 and 150 is 50 .So the greatest amount of rice or wheat that can be packed in each bag, so that each bag weighs the same, is **50 kg**.

21. . From each brand, least number of chocolates to be bought =LCM (24, 15)
LCM by prime factorisation method:
$24=2\times2\times2\times3$
$15=3\times5$
L.C.M. of 24 and $15=2\times2\times2\times3\times5=120$
[LCM of two or more numbers = product of the greatest power of each prime factor involved in the prime factorisation of all the numbers]
From each brand, Least number of chocolates to be bought =120
Hence, the number of packs of brand A=120/24=5
Number of packs of brand B=120/15=8

22. Prime Factorization:
1. **80**: $2^4\times5^1$
2. **120**: $2^3\times3^1\times5^1$
3. **100**: $2^2\times5^2$

LCM Calculation:
$LCM=2^4\times3^1\times5^2=16\times3\times25=1200$ minutes
1200 minutes is equivalent to **20 hours**.
Starting from **1:00 PM**, adding 20 hours brings us to **9:00 AM** the next day.Thus, all three movies will next start together at **9:00 AM** the next day.

5 MARKS QUESTIONS

23. We need to find the LCM.The LCM of 120, 90 and 150 is 30.
No. of pens in each bag = 120/30 =4

No. of notebooks in each bag = 90/30 =3

No. of stickers in each bag = 150/30 =5

30 gift bags can be prepared.

Each gift bag will contain **4 pens, 3 notebooks, and 5 stickers**.

24. Since the numbers are co-prime, they contain only 1 as the common factor.
Also, the given two products have the middle number in common.
So, middle number = H.C.F. of 551 and 1073 = 29;
551/29 = 19, 1073/29 = 37
Required sum = (19 + 29 + 37) = 85.

25. Find the HCF of each fruit count:
Peaches: $24= 2^3\times3^1$

Apricots: $36=2^2\times3^2$

Bananas: $60=2^2\times3^1\times5$

HCF of 24, 36, and 60: $2^2\times3^1=12$
Number of rows = 24/12 =2 , 36/12=3 , 60/12= 5
Minimum number of rows required = 2+3+5 =10
Fruits per row: Each row contains 12 fruits of one type.

CASE BASED QUESTIONS (4 MARKS EACH)

26. We need to find HCF .

$104 = 2^3 \times 13$
$96 = 2^5 \times 3$
$HCF = 2^3 = 8$

(i) Number of rows of students of Class X = 104/8 = 13

Number maximum of rows Class IX = 96/8 = 12

Total number of rows = 13 + 12 = 25

(II) No. of students of Class IX in a row = 8

No. of students of Class X in a row = 8

27. After giving 11 stamps, No. of stamps that Megha have = 143 - 11 = 132
After giving 11 stamps, No. of stamps that Abdul have = 220 - 11 = 209
The Number of equal groups that can be formed by megha's and abdul's stamps = HCF (132, 209)
To find the HCF of 132, 209 write them as product of prime factors
132 = 2 x 2 x 3 x 11
209 = 11 x 19
⇒ HCF (132, 209) = 11
The Number of equal groups that can be formed by Megha and Abdul = 11
The number of stamps in Adya's groups = 132/11 = 12
The number of stamps in Sumit's groups = 209/11 = 19

28. We need to find the HCF of 24,30 and 18.

$24 = 2^3 \times 3$

$30 = 2 \times 3 \times 5$

$18 = 2 \times 3^2$

HCF = 2 x 3 =6

Mrs. Latha can divide each class into groups of 6 students.

The second class will have 30/6 =5

29. We need to find the LCM of 5,8 and 10.

LCM of 5,8 and 10 = 40

All bulbs will blink together every 40 seconds.

Convert 1 hour to seconds:

1 hour=60 minutes=60×60=3600 seconds

Number of times=3600/40 = 90

They will blink together **90 times** in 1 hour

30. (i) √3 , π √3 /2 , e

(ii)Always. The only time V is not rational is when $r = 0$ which means that the sphere does not exist. Multiplying π by any other positive number is an irrational number.

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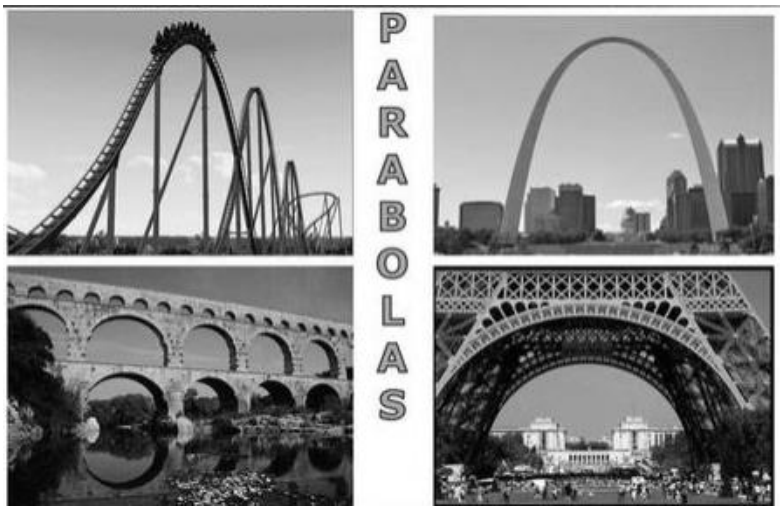
CHAPTER 2- POLYNOMIALS

MULTIPLE CHOICE QUESTIONS

1. A parabolic arch is an arch in the shape of a parabola



In structures, their curve represents an efficient method of load, and so can be found in bridges and **in** architecture in a variety of forms. Based on the above information, answer the following questions. In the standard form of quadratic polynomial, $\mathbf{ax^2 + bx + c}$, a, b and c are (AO1)

(a) All are real numbers

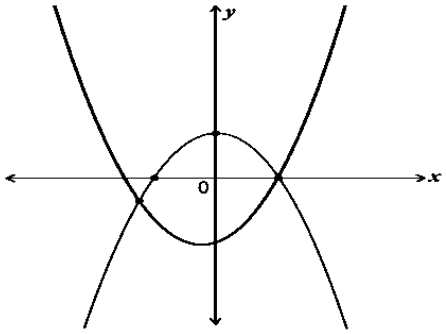
(b) All are rational numbers.

(c) a is a non-zero real number, b and c are any real numbers

(d) All are integers.

2. Two polynomials are shown below. The number of zeroes common to both the polynomials is (AO2)

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4



3. If $p(x) = x^2 + 5x + 2$, then the value of $p(3) + p(2) + p(0)$ is: (AO1)

(a) 40 **(b)** 44 **(c)** 8 **(d)** 42

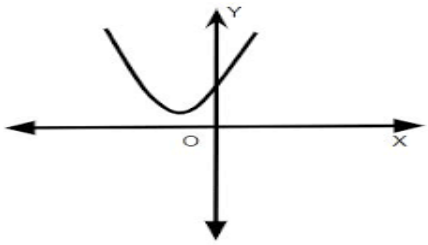
4. The zeroes of the quadratic polynomial $x^2 + 43x + 222$ are: (AO1)

(**a**) both equal (**b**) one positive one negative
(**c**) both negative (**d**) both positive

5. The graph of $x^2 + 1 = 0$ (AO1)

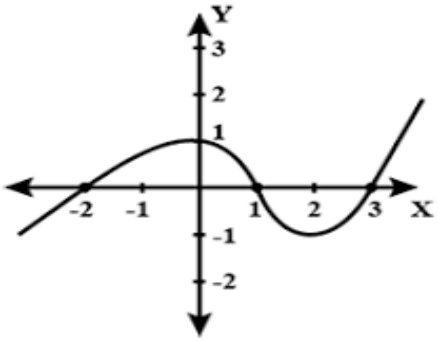
(**a**) Intersects X- axis at two distinct points. (**b**)Touches X-axis at a point.
(**c**) Neither touches nor intersects X-axis (**d**) Either touches or intersects X-axis

6. The number of zeroes of the polynomial P(x) is (AO1)



(**a**) 1 (**b**) 2 (**c**) 3 (**d**) 0

7. The zeroes of the polynomial are (AO2)



(a) 1 **(b)** 1 and -1 **(c)** -2 **(d)** – 2 , 1 and 3

8. If one zero of the polynomial $f(x)= (k^2+4)\, x^2+13x+4k$ is the reciprocal of the other, k = **(AO2)**

(a) 2 (b)-2 (c)1 (d)-1

ASSERTION AND REASON QUESTIONS

(a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A).
(b) Both Assertion(A) and Reason(R) are true but the Reason(R) is **not** the correct explanation of Assertion(A).
(c) Assertion(A) is true, but Reason(R) is false.
(d) Assertion(A) is false, but Reason(R) is true

9. Assertion: The zeroes of the polynomial p(x)=(x−1)(x−2)(x−3) are 1,2 and 3 (AO1)
Reason: The zeroes of a polynomial are the x-coordinates of the points where the graph of polynomial intersects or touches x-axis or the points on the graph where p(x)=0 .(AO1)

10. Assertion: The sum and product of the zeroes of a quadratic polynomial are – ¼ and ¼ respectively. Then the quadratic polynomial is $4x^2 + x + 1$ (AO1)
Reason: The quadratic polynomial whose sum and product of zeroes are given is x^2 + (Sum of zeroes) x + product of zeroes.

11. Assertion: $x^3 + x$ has only one real zero (AO2)

Reason: A polynomial of nth degree must have n real zeroes.

12. Assertion: The graph of linear polynomial intersects the x-axis at one point. (AO1)

Reason: For polynomial P(x) of degree n, the graph of y = P(x) intersect x-axis at most n points.

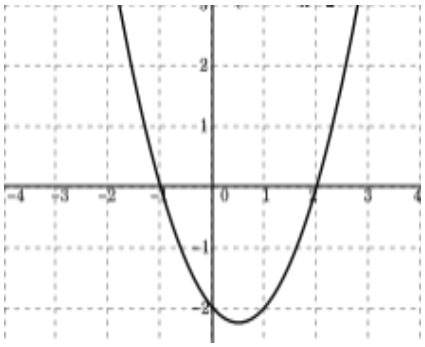
2 MARKS QUESTIONS

13. Find the quadratic polynomial whose sum and product of zeroes are √2 and $\frac{1}{3}$ respectively.(AO1)

14. If α and β are the zeros of the quadratic polynomial $2x^2 - 8x + 5$, find the value of $(\alpha + \frac{1}{\beta}) \times (\beta + \frac{1}{\alpha})$ (AO2)

15. The graph of polynomial passes through (6,0), (0,-2) and (-1,0). Write two factors of the polynomial. (AO2)

16. Form the quadratic polynomial for the graph shown in the fig (AO2)



17. If the sum of the zeroes of the quadratic polynomial $f(t) = kt^2 + 2t + 3k$ is equal to their product, find the value of k. (AO1)

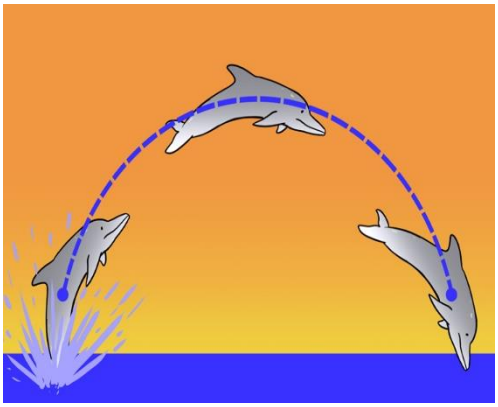
3 MARKS QUESTIONS

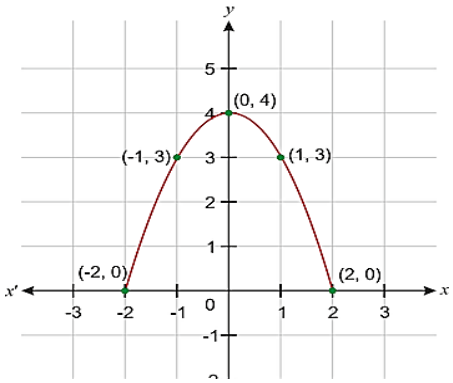
18. The demand D and supply S for a product are given by the polynomial equations:
$D(p) = 30 - 3p$ and $S(p) = 2p + 20$ where p is the price in dollars. Find the equilibrium price and demand price where supply equals demand. (AO2)

19 The population of a certain species of fish in a lake can be modeled by a cubic polynomial:
$P(t) = -2t^3+15t^2+30t+100$. What is the population at year 5 ? (AO1)

20. Find the zeroes of the polynomial $4x^2 - 3x - 1$ by factorization method and verify the relation between the zeroes and the coefficient of the polynomial. (AO1)

21. The path of a jump of a Dolphins is represented by the graph below. (AO2)





(i) Name the shape of the graph
(ii) Form the quadratic polynomial represented in the graph.

22. A teacher after teaching the chapter polynomial in class 10th, wrote the sum and product of zeros of a polynomial as 3 and -5 respectively on the blackboard to test the skill grasped by his students. Find out the Polynomials that the teacher has in her mind. (AO2)

5 MARKS QUESTIONS

23.If one zero of the polynomial $(k + 1)x^2 - 5x + 5$ is multiplicative inverse of the other, then find the zeroes of $kx^2 - 3kx + 9$, where k is constant. (AO2)

24.If α, β are zeros of the quadratic polynomial $f(x) = 2x^2 + 11x + 5$, (AO2)

find a) $\alpha^4 + \beta^4$ b) $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

25. A quadratic polynomial ax^2+bx+c is a perfect square trinomial if it can be written as $(p x + q)^2$, where p and q are constants. Show that the quadratic polynomial $g(x) = 2x^2 - 4x + 2$ is a perfect square trinomial. Find the value of x for which $g(x) = 0$. (AO2)

CASE BASED QUESTIONS (4 MARKS EACH)

26. A ball is thrown vertically upward with an initial velocity of 20 m/s from a height of 25 m. The height h of the ball at any time t seconds is given by the equation: (AO2)

$$h(t) = -5t^2 + 20t + 25$$

(i) What is the height of the ball after 3s?
(ii) At what time will the ball hit the ground.

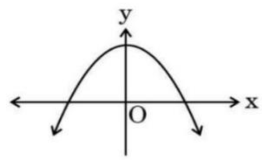
27. The height H of a roller coaster above the ground, in meters, is modeled by the quadratic polynomial: $H(x) = -3x^2 + 12x + 15$ where x is the horizontal distance from the starting point in meters. (AO2)

(i) Find the distance from the starting point when the roller coaster is at ground level.
(ii) What is the height of the roller coaster at a distance of 2m from the start.

28. Anju is designing a rectangular garden with a path around it. The area of the garden is given by the quadratic polynomial $A(x) = x^2+10x + 24$ where x is the width of the path. (AO2)

(i) What is the area of the garden when the path is 3 meters wide?
(ii) If the total area is 195 square meters, what are the possible widths of the path?

29. Rainbow is an arch of colours that is visible in the sky after rain or when water droplets are present in the atmosphere. The colours of the rainbow are generally, red, orange, yellow, green, blue, indigo and violet. Each colour of the rainbow makes a parabola. We know that any quadratic polynomial $\mathbf{p(x)=ax^2+bx+c(a\neq 0)}$ represents a parabola on the graph paper.



Based on the above, answer the following questions :

i. The graph of a rainbow ***y=f(x)*** is shown in the figure.Write the number of zeroes of the curve.

ii. If the graph of a rainbow does not intersect the ***x***-axis but intersects ***y***-axis at one point, then how many zeroes will it have?

iii. If a rainbow is represented by the quadratic polynomial $p(x)=x^2+(a+1)x+b$, whose zeroes are 2 and -3 , find the value of a and ***b***.

30.A water fountain creates a parabolic stream of water. The height of the water stream in meters is described by the equation $h(x) = -2x^2 + 12x$ where x is the horizontal distance from the fountain. (AO2)

i) At what horizontal distances does the water hit the ground?
ii) What is the height of the water stream 2 meters from the fountain?

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SOLUTIONS: CHAPTER 2- POLYNOMIALS

1. c) a is a non- zero real number, b and c are any real numbers
2. (**a**) 1
3. b) 44
4. c) both negative
5. (**c**) Neither touches nor intersects X-axis
6. (**d**) 0
7. **(d)** – 2 , 1 and 3
8. **(a)** 2

ASSERTION AND REASON QUESTIONS

9. (a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A).
10. (c) Assertion(A) is true, but Reason(R) is false.
11. c) Assertion(A) is true, but Reason(R) is false.
12. (a) Both Assertion(A) and Reason(R) are true and the Reason(R) is the correct explanation of Assertion(A).

SHORT ANSWER TYPE(2 MARKS EACH)

13. Sum of zeroes = α + β =√2.
    Product of zeroes = α β = 1/3.
    ∴ If α and β are zeroes of any quadratic polynomial, then the standard form of the polynomial is; $x^2 - (\alpha + \beta)x + \alpha\beta$
    The polynomial $= x^2 - \sqrt{2}x + \frac{1}{3} = 3x^2 - 3\sqrt{2}x + 1$
14. α and β are the zeroes of the quadratic polynomial $2x^2 - 8x + 5$

$$\alpha + \beta = \frac{-b}{a} = -\left(\frac{-8}{2}\right) = 4 \text{ and } \alpha\beta = \frac{c}{a} = \frac{5}{2}$$

$$\alpha + \frac{1}{\beta} = \frac{\alpha\beta+1}{\beta} \text{ and } \beta + \frac{1}{\alpha} = \frac{\alpha\beta+1}{\alpha}$$

$$\therefore \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \left(\frac{\alpha\beta+1}{\beta}\right)\left(\frac{\alpha\beta+1}{\alpha}\right) = \frac{(\alpha\beta+1)^2}{\alpha\beta} = \frac{\left(\frac{5}{2}+1\right)^2}{\frac{5}{2}} = \frac{\left(\frac{7}{2}\right)^2}{\frac{5}{2}} = \frac{49}{10}$$

15. Given that the polynomial passes through the points (6,0) , (0,-2) and (-1,0)

    ∴ zeros of the polynomial are x= 6 and x = -1

    ∴ the factors of the polynomial corresponding to these zeros are:(x- 6) and (x+1)

16. Here the graph of the polynomial intersect at -1 and 2
    ∴ zeros of the polynomial are α = -1 and β = 2
    the standard form of the polynomial is; $x^2 - (\alpha + \beta)x + \alpha\beta$
    $= x^2 - (-1 + 2)x + -2$

$$= x^2 - x - 2$$

17. Given f(t) = $kt^2 + 2t + 3k$

$\therefore \alpha + \beta = \frac{-b}{a} = \frac{-2}{k}$ and $\alpha\beta = \frac{c}{a} = \frac{3k}{k} = 3$

Given that sum of zeroes = product of zeroes

$$\therefore \frac{-2}{k} = 3$$

$$k = \frac{-2}{3}$$

SHORT ANSWER TYPE (3 MARKS EACH)

18. Given Demand, D(p)= 30 – 3p

Supply, S(p)=2p+20

Demand = Supply

30– 3p = 2p+ 20

30-20-3p-2p=0

10-5p=0

p = 2

equilibrium price = 2 (substitute in demand)

then D(p)=24

19. Given: P(t) = $-2t^3+15t^2+30t+100$.

Let t = 5

Then P(t)= $-2 \times 125 + 15 \times 25 + 30 \times 5 + 100$

$= -250 + 375 + 150 + 100 = 375$

The population at year 5 =375

20. We have p(x)= $4x^2 - 3x - 1$

$=4x^2 - 4x + x - 1$

$=4x(x-1) + 1(x-1)$

$=(4x+1)(x-1)$

$4x^2 - 3x - 1 = (4x+1)(x-1)$

The zeroes are $\alpha = \frac{-1}{4}$ and $\beta = 1$

Sum of zeroes = $\frac{-1}{4} + 1 = \frac{3}{4} = -\frac{(-3)}{4} = \frac{-(coefficient\ of\ x)}{coefficient\ of\ x^2}$

Product of zeroes = $\frac{-1}{4} \times 1 = \frac{-1}{4} = \frac{constant\ term}{coefficient\ of\ x^2}$

21. i) parabola

ii) Here the curve intersects X-axis at -2 and 2

$\therefore x = 2\ and\ x = -2$ are the two zeroes

$$\alpha = 2\ and\ \beta = -2$$

$\alpha + \beta = 2 + (-2) = 0$ and $\alpha\beta = -4$

$\therefore$the required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 4$

22. Given $\alpha + \beta = 3\ and\ \alpha\beta = -5$

$\therefore$the required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 3x - 5$

<u>LONG ANSWER TYPE (5 MARKS EACH)</u>

23. We have P(x)= $(k + 1)x^2 - 5x + 5$ and Q(x)= $kx^2 - 3kx + 9$

Let $\alpha\ and\ \frac{1}{\alpha}$ be the zeroes of the polynomial

So, product of zeroes = $\alpha \times \frac{1}{\alpha} = \frac{5}{(k+1)}$

$\therefore 1 = \frac{5}{(k+1)}$

(k+1)=5

K=4

$$\therefore Q(x) = 4x^2 - 12x + 9$$

$$4x^2 - 12x + 9 = 4x^2 - 6x - 6x + 9$$

$$=2x(2x - 3) - 3(2x - 3)$$

$$=(2x - 3)(2x - 3)$$

It has two equal zeroes that is , x= $\frac{3}{2}$

24. A) Given f(x) = $2x^2 + 11x + 5$

$\alpha + \beta = \frac{-11}{2}$ and $\alpha\beta = \frac{5}{2}$

$$\alpha^4 + \beta^4 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$$

$$= \left[\left(\frac{-11}{2}\right)^2 - 2 \times \frac{5}{2}\right]^2 - 2 \times \left(\frac{5}{2}\right)^2$$

$$= \left[\frac{121}{4} - 5\right]^2 - 2 \times \frac{25}{4}$$

$$= \left(\frac{101}{4}\right)^2 - \frac{25}{2} = \frac{10001}{16}$$

B) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - 2\alpha\beta$

$= \frac{\frac{-11}{2}}{\frac{5}{2}} - 2 \times \frac{5}{2}$

$= \frac{-11}{5} - 5 = -\frac{-36}{5}$

25. If the polynomial is a perfect square then $b^2 = 4ac$

Given g(x) = $2x^2 - 4x + 2 = x^2 - 2x + 1$

∴g(x) is a perfect square trinomial.

If g(x)=0

Then $x^2 - 2x + 1 = 0$

$$(x-1)^2 = 0$$

$$(x-1) = 0$$

$$x = 1$$

CASE STUDY QUESTIONS

26. i) $h(t) = -5t^2 + 20t + 25$

$h(3) = -5(3)^2 + 20(3) + 25 = -45 + 60 + 25 = 40m$

ii) h (t) = 0

$-5t^2 + 20t + 25 = 0$

$t^2 - 4t - 5 = 0$

$(t-5)(t+1) = 0$

$t = 5$ or $t = -1$

Required time, t = 5s

27. i) Given : $H(x) = -3x^2 + 12x + 15$

If the roller coaster is at ground level, then H(x) = 0

$-3x^2 + 12x + 15 = 0$

$x^2 - 4x + 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ and $x = -1$

Since x is a distance choose x = 5

ii) Given x = 2m

∴ H (2) =$-3(2)^2$ +12(2) +15

= -12 +24 +15 =27

28. i) Given A(x) = x^2+10x + 24, where x is the width of the path

Area of the park if width, x= 3

A (3) = $(3)^2$+10(3) + 24 = 9 + 30 + 24 = 63

Area of the park is 63sq. unit

ii) Given A(x) = 195

x^2+10x + 24 = 195

x^2+10x -171 = 0

x = -19 and x = 9

∴ Width, x =9

29. i) has 2 zeroes

ii). Has no zeroes

iii) let $\alpha = 2$ and $\beta = -3$

$\alpha + \beta = 2 + (-3) = -1$ and $\alpha\beta = 2 \times (-3) = -6$

$-\frac{(a+1)}{b} = -1$ and $b = -6$

$a + 1 = b$

$a + 1 = -6$

$a = -7$ and b= -6

30. i) Given equation is h(x) = $-2x^2 + 12x$,where x is the horizontal distance from the fountain

If h(x) = 0

Then, $-2x^2 + 12x = 0$

$x^2 - 6x = 0$

$x^2 = 6x$

x = 6

ii) let x = 2

Then h(2) =$-2(2)^2 + 12(2)$

= -8 + 24= 16

****

CHAPTER 3: PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

MULTIPLE CHOICE QUESTIONS

01. Given below are a pair of linear equations in two variables. **(AO2)**

$4x + 2y = 18$ and $3x - 6y = 8$

Which of the following pairs of equations have the same number of solution(s) as the given pair?

(a) $3a + 3b = 18$; $a + b = 6$ (b) $a - b = 4$; $b - a = 4$

(c) $6a - 2b = 10$; $3a + b = 5$ (d) $7a + 9b = 27$; $28a + 36b = 76$

02. Given below are three equations; a pair of them have infinite solutions.

Find the pair among the three equations given below**.** **(AO1)**

i. $3x-2y=4$ ii. $6x+2y=8$ iii. $12x-8y=16$

(a) i and ii (b) i and iii (c) ii and iii (d) None of the pairs

03.The angles of cyclic quadrilaterals ABCD are: $A = (6x+10)^0$, $B=(5x)^0$, $C = (x+y)^0$ and

$D= (3y-10)^0$. The value of x and y is: **(AO2)**

(a) $x=20°$ and $y = 10°$ (b) $x=20°$ and $y = 30°$ (c) $x=44°$ and $y=15°$ (d) $x=15°$ and $y=15°$

04. If a pair of linear equations given by $a_1x + b_1y + c_1=0$ and $a_2x + b_2y + c_2=0$ has a unique

solution,then which of the following is true? **(AO1)**

(a) $a_1a_2 = b_1b_2$ (b) $a_1b_2 \neq a_2b_1$ (c) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ (d) $\frac{a_1}{b_2} \neq \frac{b_1}{a_2}$

05. One equation of a pair of dependent linear equations is $-5x + 7y = 2$ the second equation can be **(AO2)**

(a) $10x + 14y + 4 = 0$ (b) $-10x - 14y + 4 = 0$ (c) $-10x + 14y + 4 = 0$ (d) $10x - 14y = -4$

06. Two linear equations in variables x and y are given below: **(AO2)**

$a_1x+b_1y+c=0$

$a_2x+b_2y+c=0$

Which of the following pieces of information is independently sufficient to determine if a solution exists or not for this pair of linear equations?

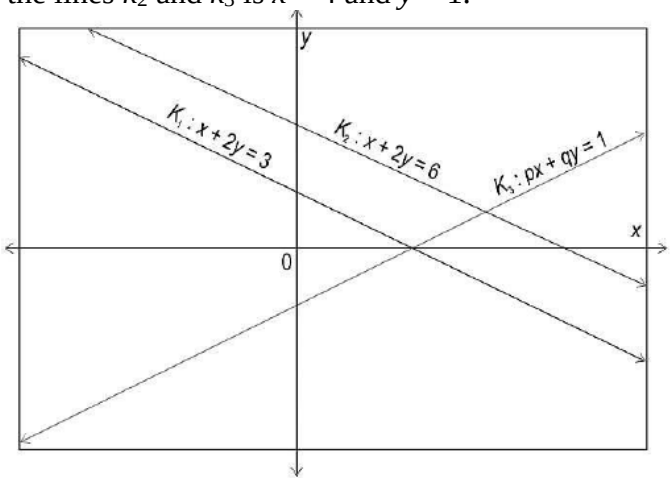
I. $a_1/b_1=a_2/b_2=1$ II. $a_1/a_2=b_1/b_2$ III. $a_1/a_2=a_1/b_1\neq 1$ IV. $a_1/a_2\neq b_1/b_2$

(a) IV only (b) I and IV (c)II and IV (d)I and III

07. In an MCQ test, a student guesses the correct answer x out of y times. If the probability that the student guesses the answer wrong is 2/3 then what is the relation between x and y . **(AO1)**

(a)y=3x (b) x =3y (c)3x=2y (d)2x=3y

08. The lines k_1, k_2 and k_3 represent three different equations as shown in the graph below. The solution of the equations represented by the lines k_1 and k_3 is x = 3 and y = 0 while the solution of the equations represented by the lines k_2 and k_3 is x = 4 and y = 1. **(AO2)**



Which of these is the equation of the line k_3?

(a) x - y = 3 (b) x - y = -3 (c) x + y = 3 (d) x + y = 1

ASSERTION AND REASONING QUESTIONS

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

09. **Assertion:** If the system of equations 2x+3y=7 ; 2ax+(a+b)y=28 has infinitely many solutions, then 2a- b=0 **(AO1)**

Reason: The system of equations 3x–5y =9 and 6x–10y =8 has infinitely many solutions

10. **Assertion:** 2x + 3y is a linear equation in two variables **(AO1)**

Reason: General form of a linear equation in two variables is $ax^2 + bx + c = 0$

11. **Assertion:** The graph of the linear equations 3x+2y=12 and 5x-2y=4 gives a pair of intersecting lines. **(AO1)**

Reason: The graph of linear equations a1x+b1y+c1=0 and a2x+b2y+c2=0 gives a pair of intersecting lines if a1/a2 ≠ b1/b2

12. **Assertion:** The number of common solutions for the system of linear equations 5x+4y+6=0 and 10x+8y=12 is zero. **(AO2)**

Reason: The graph of linear equations a1x+b1y+c1=0 and a2x+b2y+c2=0 gives a pair of intersecting lines if a1/a2 ≠ b1/b2

SHORT ANSWER TYPE QUESTIONS :(2 MARKS EACH)

13. If 51x+49y=150 and 49x+51y=50 then obtain the value of x−y**:** x+y. **(AO1)**

14. Find the relation between p and q if x = 3 and y = 1 is the solution of the pair of equations

x – 4y + p = 0 and 2x + y – q – 2 = 0 **(AO2)**

15. Name the geometrical figure enclosed by graph of the equations x + 7 = 0, y – 2 = 0 and

x – 2 = 0, y + 7 = 0. **(AO1)**

16. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row there would be 3 rows more. Find the number of students in the class. **(AO2)**

17. Anuj had some chocolates, and he divided them into two lots A and B. He sold the first lot at the rate of ₹2 for 3 chocolates and the second lot at the rate of ₹1 per chocolate and got a total of ₹400. If he had sold the first lot at the rate of ₹1 per chocolate, and the second lot at the rate of ₹4 for 5 chocolates, his total collection would have been ₹460. Find the total number of chocolates he had.

(AO2)

IV.SHORT ANSWER TYPE QUESTIONS :(3 MARKS)

18. If 16 is subtracted from twice the greater of two positive numbers, the result is half the other number. If 1 is subtracted from half the greater number, the result is still half the other number. Find the two numbers. **(AO2)**

19. Given below is a pair of linear equations: $2x - my = 9$: $4x - ny = 9$ Find at least one pair of the possible values of *m* and *n*, if exists, for which the above pair of linear equations have:
    i) a unique solution
    ii) infinitely many solutions
    iii) no solution Show your work. **(AO1)**

20. If (6, 0) and (0, 2) are two of the points of intersections of two lines represented by a pair of linear equations. i) How many points of intersections does the pair of linear equations have in total? Justify your answer. ii) Find the equation that represents one of the lines of the above pair. Show your work. **(AO1)**

21. A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the number. **(AO2)**

22. In a competitive examination , one mark is awarded for each correct answer, while $\frac{1}{2}$ mark is deducted for every wrong answer. Rahul answered 120 questions and got 90 marks. How many questions did he answer correctly? **(AO2)**

LONG ANSWER TYPE QUESTIONS :(5 MARKS EACH)

23. For the given pair of linear equations $2x + y = 6$, $2x = y + 2$. Draw the graph of the two equations on the same graph paper. **(AO1)**

    Find the ratio of the areas of two triangles, formed by the given lines with x-axis and with the y-axis. **(AO1)**

24. If 45 is subtracted from twice the greater of two numbers, it results in the other number. If 21 is subtracted from twice the smaller number, it results in greater number. Find the numbers. **(AO2)**

25. When the son will be as old as what his father is today their ages will add upto 126 years. When the father was as old as what his son is today, their ages added upto 38 years. Find their present ages. **(AO2)**

CASE BASED QUESTIONS (4 MARKS EACH)

26. Ram and Rahim are two friends; both own rectangular plots. Ram is the owner of a rectangular plot whose perimeter is 50m and Rahim is the owner of a rectangular plot whose perimeter is 100m. Rahim's plot has a length twice that of Ram's plot and breadth is 5m more than that of Ram's plot.
Answer the following questions based on the given data:

a) Write a pair of linear equations in 2 variables for finding the dimensions of both the plots. (2 mark) **(AO1)**

b) Find the dimensions of Ram's plot (2 mark)

27. Mrs. Renu Sharma is the owner of a famous amusement park in Delhi. The ticket charges for the park is Rs 150 for children and Rs 400 for adults. Generally, she does not go to the park to check the accounts, and her team of staff manages it. One day she decided to check the accounts randomly. When she checked the cash counter, she found that 750 tickets were sold and Rs 212,500 was collected. **(AO1)**

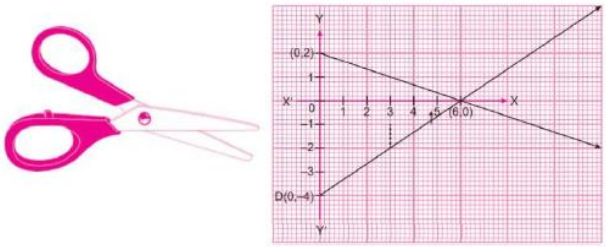
a) Represent the situation algebraically.
b) Find the number of children that visited the amusement park on that day.

c) Also, find the number of adults who visited the amusement park on the same day.

(OR)

Compute the total amount collected if 415 children and 150 adults visited the park.

28. The scissors which is so common in our daily life use, its blades represent the graph of linear equations



(i) What is the solution of the above system of equations. **(AO1)**

(ii) Write whether the system is consistent or not and give reason to support your statement.

(iii) If (1, 2) is the solution of linear equations $ax + y = 3$ and $2x + by = 12$,then find the values of a and b .

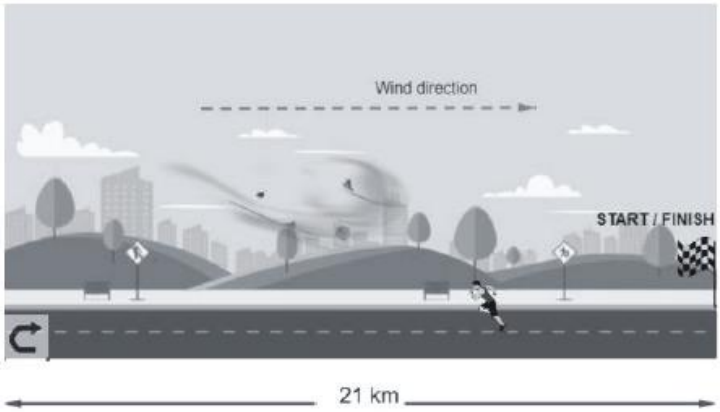
29. Seema is a taxi driver working with ABC taxi services and she has set a target to earn **Rs.1000** on Monday. The meter charges the customer at a **rate of Rs 10 per kilometre** for travelling. The customer has to pay Rs.10 per Km along with an additional fixed charge of Rs.115. But the driver **gets only half of the amount paid** by the traveller after deduction of the commission and other charges by the taxi company. **(AO2)**

(a) If x were the fixed charges and y the distance covered by a customer, then form an equation to represent Seema's share.

(b) On Monday 3 customers, separately, book the taxi for travelling 45km,33km and 55km.What is the total collection of Seema?

30. At a marathon ,the 42km route is designed such that the marathoners run in straight line for 21km and return back along the same line in the opposite direction.
A marathoner running against wind ,covered the first half distance in 2 hours.Then he covered the second half distance running with the wind in 1.5 hours.Assuming the marathoner ran at a constant speed and the wind speed and direction did not change.

(AO2)



(a) Calculate the speed of the marathoner in Km/hr.
(b) Find the speed of the wind.

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SOLUTIONS: CHAPTER 3-PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1. (c) 6a – 2b =10; 3a + b = 5

2. (b) i and iii

3. (b) x=20° and y = 30°

4. (b) $a_1b_2 \neq a_2b_1$

5. (d) $10x - 14y = -4$

6. (b) I and IV

7. (a) x - y = 3

8. (a)$y=3x$

9. option c

10. option c

11. option a

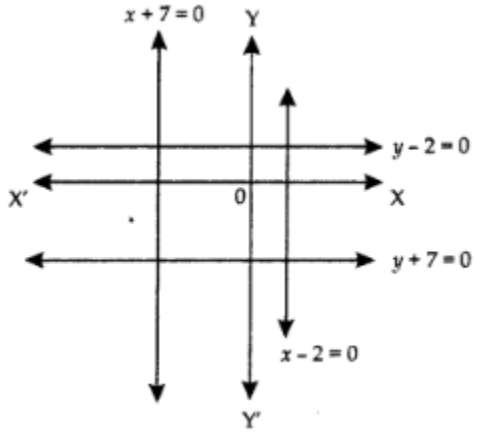
12. option b

13. Adding eq (1) & (2)

100x+100y=200
x+y=2...(3)
Subtracting eqn (2) from (1)
2x−2y=100
x−y=50...(4)
Adding eqn (3) & (4)
x+y=2
x−y=50
2x=52 (OR) x=26
x+y=2
26+y=2
y= −24
finding x−y & x+y i.e from (3)&(4)
x−y : x+y⇒50 : 2 ⇒25:1

14. Since x = 3, y = 1 is the solution of
x – 4y + p = 0 …….. (i)
2x + y – q – 2 = 0 …… (ii)
So, x = 3, y = 1 must satisfy both (i) and (ii).
⇒ 3 – 4(1) + p = 0 ⇒ p = 1.
and 2(3) + (1) – q – 2 = 0 ⇒ q = 5
Since, 1 = – 4 + 5
∴ p = 1 = – 4 + 5
= q – 4
⇒ p = q – 4
Thus, p is lesser than q by 4.

15. A square



16. Let number of rows is x and number of students is y in a row.

Then total number of students = xy

(x−2)(y+1)=xy

=> x-2y =2

$(x+3)(y-1)=xy$

=> $-x+3y=3$

On Solving, we get

$x=12$,$y=5$

17.

Let the number of chocolates in lot A be x

And let the number of chocolates in lot B be y

∴ Total number of chocolates = x + y

Price of 1 chocolate = ₹ $\frac{2}{3}$, so for x chocolates = $\frac{2}{3}$x and price of y chocolates at the rate of ₹ 1 per chocolate = y.

∴ By the given condition $\frac{2}{3}x + y = 400$

⇒ 2x + 3y = 1200 .....(i)

Similarly $x + \frac{4}{5}y = 460$

⇒ 5x + 4y = 2300 .....(ii)

By solving (i) and (ii) we get

x = 300 and y = 200

∴ x + y = 300 + 200 = 500

So, Anuj had 500 chocolates.

18. Let the two numbers be x and $(x>y)$

$\therefore 2x-16=12y \Longrightarrow 4x-y=32\ldots(1)$

and $12x-1=12y \Longrightarrow x-y=2\ldots(1)$

Solving, we get $x=10$ and $y=8$

Hence the two numbers are 10 and 8

19. Any correct values that satisfy the conditions

20. i) Writes that the pair will have infinitely many solutions. Reasons that as there are more than one points of intersection, the pair is of coincident or overlapping lines.

ii) Substitutes the values of the point of intersection (6, 0) in the equation of a line ax + by = c as: 6a + 0 = c or a = $c/6$

Substitutes the values of the second point of intersection (0, 2) in the equation as: 2b = c

or b = $c/2$

Rewrites the equation of a line by substituting the values of a and b in terms of c as: *c*/6x + /2 y = c
Simplifies the above equation by taking c = 1 to find the equation of the line as

x + 3y = 6.

21. Let the two – digit number = 10x + y

Case I: multiplying the sum of the digits by 8 and then subtracting 5 = two – digit number

8x + 8y – 5 = 10x + y

2x – 7y = - 5...(i)

Case II: Multiplying the difference of the digits by 16 and then adding 3 = two – digit number

16x – 16y + 3 = 10x + y

6x – 17y = - 3...(ii)

Now, multiplying Eq. (i) by 3 and then subtracting from Eq. (ii), we get

6x – 17y = - 3

6x – 21y = - 15

4y = 12

y = 3

Now put the value of y in Eq. (i) we get

2x – 7 × 3 = - 5

2x = 21 – 5 = 16 .

So, x = 8

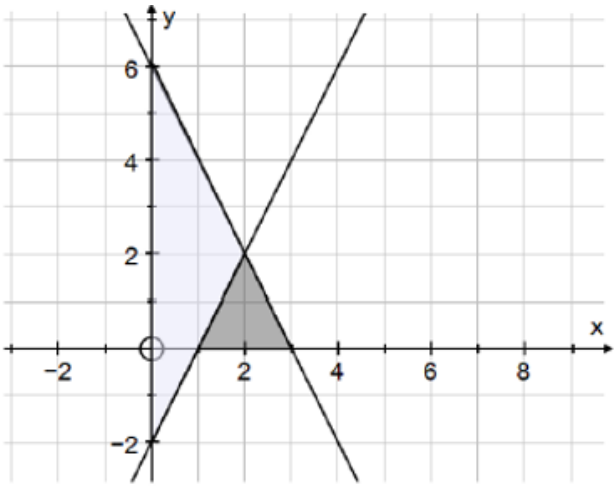
Hence, the required two – digit number

= 10 × 8 + 3 = 80 + 3 = 83

22. Let the number of right answers and wrong answers be *x* and *y* respectively.
According to the given information,
x + y = 120---------(1)
Also, $1x - \frac{1}{2}y = 90$ ---------(2)
Solving 1 and 2, we obtain *x* = 100 and y=20
Therefore, number of right answers = 100
And number of wrong answers = 20

23. Vertices of △ABC are A(2,2), B(1,0) and C(3,0)
Vertices of △ADE are A(2,2),D(0,6) and E(0,-2)
Area of △ABC = 2 sq units
Area of △ADE= 8 sq units
Ratio areas of two triangles formed by the lines representing these equations with the x-

axis and lines with the y-axis. = 2:8 =1:4



24. Let x, y (x > y) be the required numbers.

First condition: According to it:

$2x - 45 = y$

$\Rightarrow 2x - y - 45 = 0$ …… (i)

Second condition: According to it:

$2y - 21 = x$

$\Rightarrow x - 2y + 21 = 0$ …… (ii)

Multiplying (i) by 2 and subtracting from (ii), we get

$(x - 2y + 21) - 2(2x - y - 45) = 0$

$\Rightarrow x - 2y + 21 - 4x + 2y + 90 = 0$

$\Rightarrow -3x + 111 = 0$

$\Rightarrow x = 111/3 = 37$

Putting x – 37 in (i), we get

$2 \times 37 - 45 = y$

$\Rightarrow y = 74 - 45$

$\Rightarrow y = 29$

∴ The required numbers are 37, 29.

25. Let son's present age be x years and father's present age be y years.

Difference in father's and son's age = (y – x) years

son's age will be [x – (y – x)] years.

Thus, Condition I:

$\Rightarrow [x + (y - x)] + [y + (y - x)] = 126$

$\Rightarrow 3y - x = 126$

$\Rightarrow x - 3y = -126$ ……. (i)

Condition II:

$\Rightarrow [y - (y - x)] + [x - (y - x)] = 38$

$\Rightarrow 3x - y = 38$ …… (ii)
(i) + (ii) $\Rightarrow 4(x - y) = -88$
$\Rightarrow x - y = -22$ ……. (iii)
(ii) – (i) $\Rightarrow 2(x + y) = 164$
$\Rightarrow x + y = 82$ …….. (iv)
(iii) + (iv) $\Rightarrow 2x = 60$
$\Rightarrow x = 30$
(iv) – (iii) $\Rightarrow 2y = 104$
$\Rightarrow y = 52$
Hence, present age of father is 52 years and that of son is 30 years.

26. let x m be the length and y m be the breadth of Ram's plot and 2x m be the length, and (y+5) m be the breadth of Rahim's plot
Apply the formula of the perimeter of the rectangle
2(x+y) =50 and 2(2x+y+5) =100
x+y =25…..(1)
2x+y = 45 ……. (2)

x= 20, y=5

Ram's plot Length =20m , breadth= 5m

27. Let the number of children visited be *x* and the number of adults visited be *y*
Since 750 people visited, $x + y = 750$.
Collected amount is Rs 212500 thus
$150x + 400y = 212500$
$3x + 8y = 4250$
Multiplying x + y= 750 by 3 and solving we get
Number of children = 350
Number of adults = 400

415 x 150 + 150 x 400
= 62250 + 60000 = Rs 122250

28. i) Sol : (6, 0)
ii) consistent because there is solution
iii) a= 1 ,b= 5

29. (a) $\frac{x+10y}{2}$ (b) Rs.1675

30. m-w=21/2

m+w=21/1.5

getting m=49/4=12.25 km/hr

w= 7/4 =1.75km/hr

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CHAPTER 4-QUADRATIC EQUATIONS

MULTIPLE CHOICE QUESTION

1. Which of the following is not a quadratic equation? (A02)

(a) $2(x-1)^2 = 4x^2 - 2x + 1$

(b) $2x - x^2 = x^2 + 5$

(c) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

(d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

2. If $(x - a)$ is one of the factors of the polynomial $ax^2 + bx + c$, then one of the roots of $ax^2 + bx + c = 0$ is (AO2)

(a) 1 (b) c (c) a (d) none of these

3. Which of the following are the roots of the quadratic equation, $x^2 - 9x + 20 = 0$ by factorisation? (AO1)

(a) 3, 4 (b) 4, 5 (c) 5, 6 (d) 6, 7

4. If $(1 - p)$ is a root of the equation $x^2 + px + 1 - p = 0$, then roots are (AO2)

(a) 0, 1 (b) – 1, 1 (c) 0, – 1 (d) – 1, 2

5. Which of the following equations has no real roots? (AO1)

(a) $x^2 - 4x + 3\sqrt{2} = 0$

(b) $x^2 + 4x - 3\sqrt{2} = 0$

(c) $x^2 - 4x - 3\sqrt{2} = 0$

(d) $3x^2 + 4\sqrt{3}x + 4 = 0$

6. $(x^2 + 1)^2 - x^2 = 0$ has (AO1)

(a) four real roots (b) two real roots

(c) no real roots (d) one real root

7. If the difference of the roots of the equation $x^2 - bx + c = 0$ be 1, then

(a) $b^2 - 4c + 1 = 0$ (b) $b^2 + 4c = 0$ (c) $b^2 - 4c - 1 = 0$ (d) $b^2 - 4c = 0$

8. If the roots of $ax^2 + bx + c = 0$ are equal in magnitude but opposite in sign, then (AO2)

(a) $a = 0$ (b) $b = 0$ (c) $c = 0$ (d) none of these

\ASSERTION AND REASONING

In the following question a statement of Assertion (A) is followed by a statement of reason (R), choose the correct answer out of the following cases:

(a) Both A and R are true and R is the correct explanation for A.
(b) Both A and R are true and R is not correct explanation for A.
(c) A is true but R is false.
(d) A is false but R is true.

9. **Assertion (A):** Sum of ages of two friends is 20 years. Four years ago, the product of their ages in years was 48. Then the difference between their ages is 16. (AO1)
Reason (R): For quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

10. **Assertion (A):** The equation $8x^2 + 3kx + 2 = 0$ has equal roots than the value of k is $\pm \frac{8}{3}$.
Reason (R): The equation $ax^2 + bx + c = 0$ has equal roots if $D = b^2 - 4ac = 0$. (AO1)

11. **Assertion (A)**: The degree of quadratic equation is always 2. Hence, $x^2 - 1 = 0$ is pure quadratic equation.
Reason (R): An equation of the form $ax^2 + c = 0$ is known as pure quadratic equation. (AO1)

12. **Assertion (A):** The product of two successive positive integers multiples of 5 is 300, then the two numbers are 15 and 20.
Reason (R): The product of two consecutive integers is a multiple of 2. (AO1)

VERY SHORT QUESTIONS(2 MARKS EACH)

13. Solve the following quadratic equation by factorisation: (AO1)

$12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

14. Two numbers differ by 3 and their product is 504. Find the numbers. (AO1)

15. Solve the following equation using by quadratic formula: (A01)

$x^2 + 5x + 5 = 0.$

16. Solve the following equation by using quadratic formula: (AO1)

$9x^2 - 12x + 4 = 0.$

17. Solve the following equation by using quadratic formula: (AO1)

$x^2 + x + 1 = 0$

SHORT QUESTIONS(3 MARKS EACH)

18. Using quadratic formula, solve the following quadratic equation for x: (AO2)

$p^2x^2 + (p^2 - q^2)x - q^2 = 0.$

19. Solve for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$; $a \neq 0, b \neq 0, x \neq 0$ (AO2)

20. Solve for x: $\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}$, ($x \neq 2,4$) (AO2)

21. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hour from its usual speed. Find the usual speed of the train. (AO2)

22. A two-digit number is such that product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number. (AO2)

LONG QUESTIONS(5 MARKS EACH)

23. Solve for x: [1/(x+1)] + [3/(5x+1)] = 5/(x + 4); x = -1, $-1/5$, -4 (AO2)

24. Find the discriminant of the equation $3x^2 - 2x + 1/3 = 0$ and hence find the nature of its roots. Find them, if they are real. (AO1)

25 The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field. (AO2)

CASE BASED STUDY QUESTIONS

26. Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km. (AO2)

(a). What will be the distance covered by Ajay's car in two hours?
(b). Which of the following quadratic equation describe the speed of Raj's car? (c). What is the speed of Raj's car?
(d). How much time took Ajay to travel 400 km?
(i) 20 hour (ii) 40 hour (iii) 25 hour (iv) 16 hour

27. Quadratic equations has many aspects in real life. Quadratic equations are very useful while calculating areas of a room, box, perimeter a plot of land etc. For example Rajuvinder need to make a top of a study table whose length is 1.5 m longer than its breadth. He has various questions in his mind. Help him to answer the questions. (AO2)

(a) If Rajuvinder has $3m^2$ of plywood. He used it to make a top of a study table whose length is 1.5 m longer than its breadth. Represent this situation in form of quadratic equation.
(b) What are the dimensions of top of the study table?
(c) What are the dimensions of square shaped study table?

28. The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream (AO2)

(a) Let speed of the stream be x km/hr. then speed of the motorboat in upstream will be
(b) What is the relation between speed, distance and time?

(c) Which is the correct quadratic equation for the speed of the current?

29. If p(x) is a quadratic polynomial then p(x) = 0 is a quadratic equation. Hence $p(x) = ax^2 + bx + c = 0$ is known as quadratic equation. The roots of the quadratic equation $ax^2 + bx + c = 0$ and zeros of the quadratic polynomial $ax^2 + bx + c$ are same. The roots of the quadratic equation is obtained by solving the quadratic by factorisation and by using quadratic formula or discriminant method. The roots of quadratic equation may be equal or unequal or not real

Answer the questions based on above (AO2)

(a) The roots of the quadratic equation $\sqrt{3}x^2 + 8x + 5\sqrt{3} = 0$ by factorisation are
(i) $-\sqrt{3}, -5\sqrt{3}$ (ii) $-\sqrt{3}, \frac{-5}{\sqrt{3}}$ (iii) $\sqrt{3}, \frac{5}{\sqrt{3}}$ (iv) $\sqrt{3}, 5\sqrt{3}$

(b) The roots of the quadratic equation $100x^2 - 20x + 1 = 0$ are
(i) unequal (ii) equal and real (iii) equal but not real (iv) both zeroes

(c) $x^4 + 1 + x^2 = 0$, has
(i) four real roots (ii) two real roots (iii) no real roots (iv) one real root

30. Two schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey Rs. x per student and Cricket Rs. y per student. School 'P' decided to award a total of Rs.9,500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award Rs.7,370 for the two games to 4 and 3 students respectively.

Based on the given information, answer the following questions: (AO2)

(i) Represent the following information algebraically (in terms of x and y)

(ii) What is the prize amount for hockey?

(OR)

Prize amount on which game is more and by how much?

(iii) What will be the prize amount if there are 2 students, each from two games?

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SOLUTIONS: CHAPTER 4-QUADRATIC EQUATIONS

MULTIPLE CHOICE QUESTION

1.(c) $2x^2 + 3 + 2\sqrt{6}x + x^2 = 3x^2 - 5x$

$2\sqrt{6}x + 5x + 3 = 0$

2. (c) $\because$ x – a is one of the factors

$\therefore$ one root = a.

3. (b) Given equation is $x^2 - 9x + 20 = 0$

$\Rightarrow x^2 - 5x - 4x + 20 = 0$

$\Rightarrow x(x - 5) - 4(x - 5) = 0$

$\Rightarrow (x - 5)(x - 4) = 0$

$\Rightarrow$ either $x - 5 = 0$ and $x - 4 = 0$

$\Rightarrow x = 5$ and $x = 4$ $\therefore$ x = 4 and 5 are the roots/solution of the given quadratic equation.

4. (c) (1 – p) is a root

$\therefore (1 - p)^2 + p(1 - p) + 1 - p = 0$

$\Rightarrow (1 - p)[1 - p + p + 1] = 0$

$\Rightarrow (1 - p)(2) = 0$

$\Rightarrow p = 1$ $x^2 + x = 0$

One root = 0 and another root = – 1

$\therefore$ roots are 0 and –1.

5. (a) $D < 0$

6. (c) No real roots

7. (c) Let roots are α and β

$\Rightarrow \alpha - \beta = 1$

$\because (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow 1 = \beta^2 - 4c$

$\Rightarrow \beta^2 - 4c - 1 = 0$

8. (b) $\because$ sum of roots = 0

$\Rightarrow b/a = 0$

$\Rightarrow b=0$

ASSERTIONS AND REASON QUESTIONS

9. (d) 10. (a) 11. (a) 12. (b)

2 MARKS QUESTIONS

13. $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

$\Rightarrow 12abx^2 - 9a^2x + 8b^2x - 6ab = 0$

$\Rightarrow 3ax(4bx - 3a) + 2b(4bx - 3a) = 0$

$\Rightarrow (4bx - 3a)(3ax + 2b) = 0$

$\Rightarrow 4bx - 3a=0$ or $3ax + 2b$ $\Rightarrow x=3a/4b$ or $x= -2b/3a$

14. Let one number be x

$\therefore$ Other number be $x + 3$ ATQ $x(x + 3) = 504$

$\Rightarrow x^2 + 3x - 504 = 0$

$\Rightarrow x^2 + 24x - 21x - 504 = 0$

$\Rightarrow (x + 24)(x - 21) = 0$

$\Rightarrow x = -24$ or $x = 21$

$\Rightarrow$When $x = -24$, numbers are -24 and $-24 + 3 = -21$

$\Rightarrow$When $x = 21$, numbers are 21 and $21 + 3 = 24$.

15. Given equation is $x^2 + 5x + 5 = 0$

$\Rightarrow$Here $a = 1$, $b = 5$, $c = 5$.

$\Rightarrow D = b^2 - 4ac$

$\Rightarrow D = (5)2 - 4 \times 1 \times 5$

$\Rightarrow D = 5 > 0$

$\therefore$ Solution is given by, $x = \frac{-b \pm \sqrt{D}}{2a}$

$x = \frac{-5+\sqrt{5}}{2\times1}$, $\frac{-5-\sqrt{5}}{2\times1}$

$\Rightarrow x = \frac{-5+\sqrt{5}}{2}$, $\frac{-5-\sqrt{5}}{2}$ are solutions.

16. Given equation is 9x 2 – 12x + 4 = 0

Here a = 9, b = – 12, c = 4

$D = b^2 - 4ac$

$D = (-12)2 - 4 \times 9 \times 4$ $D = 144 - 144 = 0$

D = 0 Equation has equal roots given by x = – b/2a

$\Rightarrow x = -(-12)/(2\times9) = 12/18 = 2/3$

$\therefore x = 2/3$ is the required solution.

17. Given equation is $x^2 + x + 1 = 0$

Here, a = 1, b = 1, c = 1

$D = b^2 - 4ac$

$D = (1)2 - 4 \times 1 \times 1$

$D = -3 < 0$

$\therefore$ given quadratic equation has no solution

<u>3 MARKS QUESTIONS</u>

18. $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Here $a = p^2$, $b = (p^2 - q^2)$, $c = -q^2$,

$D = b^2 - 4ac = (p^2 - q^2)^2 - 4 \times p^2 \times (-q^2) = (p^2 + q^2)$

Now $x = \frac{-b \pm \sqrt{D}}{2a}$

$X = [-(p^2 - q^2) + \sqrt{(p^2 + q^2)^2}]/ 2 \times p^2$; $x = [-(p^2 - q^2) - \sqrt{(p^2 + q^2)^2}]/ 2 \times p^2$

$X = q^2/p^2, -1$

19. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x} \Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$= \frac{x-a-b-x}{x(a+b+c)} = \frac{b+a}{ab} \Rightarrow \frac{-(a+b)}{x(a+b+c)} \Rightarrow \frac{a+b}{ab}$

$\Rightarrow x(a + b + x) = -ab$

$\Rightarrow x^2 + (a + b)x + ab = 0$

$\Rightarrow (x + a)(x + b) = 0$

$\Rightarrow x = -a, x = -b$

20. $\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}$

$\Rightarrow \frac{(x-1)(x-4) + (x-3)(x-2)}{(x-2)(x-4)} = \frac{10}{3}$

$\Rightarrow 6x^2 - 30x + 30 = 10x^2 - 60x + 80$

$\Rightarrow 4x^2 - 30x + 50 = 0$

$\Rightarrow 2x^2 - 15x + 25 = 0$

$\Rightarrow 2x^2 - 10x - 5x + 25 = 0$

$\Rightarrow 2x(x - 5) - 5(x - 5) = 0$ $(x - 5)(2x - 5) = 0 \Rightarrow x = 5$ or $5/2$

21. Let usual speed of the train be x km/h ; Distance = 300 km ; Time taken = $\frac{300}{x}$ hours

Increased speed = (x + 5) km/h

Then time taken = $\frac{300}{x+5}$ hours

ATQ $\frac{300}{x} + \frac{300}{x+5} = 2$

$\Rightarrow 300x + 1500 - 300x = 2(x^2 + 5x)$

$\Rightarrow 2x^2 + 10x - 1500 = 0$

$\Rightarrow x^2 + 5x - 750 = 0$

$\Rightarrow (x + 30)(x - 25) = 0$ $x = -30$ rejected or $x = 25$

$\Rightarrow$ Usual speed = 25 km/h

22. Let digit at unit's place = x and digit at ten's place = y

$\therefore$ Number = 10y + x

A.T.Q. $xy = 18$ $y = \frac{18}{x}$ ... (i)

and $10y + x - 63 = 10x + y$ $9y - 9x - 63 = 0$

$y - x - 7 = 0$

$\frac{18}{x} - x - 7 = 0$ [Using eq. (i)]

$18 - x^2 - 7x = 0$

$x^2 + 7x - 18 = 0$

$(x + 9)(x - 2) = 0$

$x = -9, x = 2$

When $x = 2$, $y = \frac{18}{2} = 9$

$\therefore$ Number = 92.

5 MARKS QUESTIONS

23. Given, $[1/(x+1)] + [3/(5x + 1)] = 5/(x + 4)$; $x = -1, -1/5, -4$

Let us take the LCM of denominators and cross multiply the terms.

$[1(5x + 1) + 3(x+1)]/[(x+1)(5x+1)] = 5/(x + 4)$

$[5x+1+3x+3]/ [5x^2 + x + 5x + 1] = 5/(x + 4)$
$(8x+4)(x + 4) = 5(5x^2 + 6x + 1)$
$8x^2 + 32x + 4x + 16 = 25x^2 + 30x + 5$
$25x^2 + 30x + 5 - 8x^2 – 36x - 16 = 0$
$17x^2 - 6x - 11= 0$
$17x^2 - 17x + 11x - 11 = 0$
$17x(x-1) + 11(x - 1) = 0$
$(17x+11)(x-1) = 0$
$17x+11 = 0, \quad x-1=0$
$x = -11/17, x = 1$

24. Given,
$3x^2 - 2x+1/3= 0$
Here, a = 3, b = -2 and c = 1/3
Since, Discriminant = b^2-4ac
$= (-2)^2 - 4 \times 3 \times 1/3$
$= 4 - 4=0.$
Hence, the given quadratic equation has two equal real roots, -b/2a and – b/2a.
2/6 and 2/6 or 1/3, 1/3

25. Then, larger side of the rectangle = (x + 30) m
Diagonal of the rectangle = $\sqrt{[x^2+(x+30)^2]}$
As given,
the length of the diagonal is = x + 60 m
$\Rightarrow x^2 + (x+30)^2 = (x + 60)2$
$\Rightarrow x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$
$\Rightarrow x^2 - 60x - 2700 = 0$
$\Rightarrow x^2 - 90x + 30x - 2700 = 0$
$\Rightarrow x(x-90) + 30(x-90)$
$\Rightarrow (x-90) (x + 30) = 0$
$\Rightarrow x = 90,-30$

CASE BASED STUDY QUESTIONS

26. (a) (i) Distance covered by Ajay's car in two hours:
∵ It is given that Speed of Raj's car = X km/ h
and also given that Ajays car is 5 km/h faster
∴ Speed of Ajay's car = x + 5 km/h
We know that the the distance D in km is given by:
D = S x T
where, S = Speed in km/h
and T = time taken in hr
Hence, the distance travelled by Ajay's care in 2 hrs:
= 2 (X + 5) km/h

(b) (iii) Quadratic equation for the speed of Raj's car:

It is given that Raj took 4 hours more than Ajay to complete the journey of 400km
We know that the time T in hrs is given by:

$T = \frac{D}{S}$, where D = distance travelled in km
and S = Speed in km/h

∵ Speed of Raj's Car = X km\h
∴ Time taken to travel 400 km by Raj's car = $\frac{400}{X}$

∵ Speed of Raj's Car = X + 5 km\h
∴ Time taken to travel 400 km by Ajay's car = $\frac{400}{X+5}$
By given condition, Time taken by Raj – Time taken by Ajay = 4 hrs

∴ $\frac{400}{X} - \frac{400}{X+5} = 4$
∴ $400 \left(\frac{1}{X} - \frac{1}{X+5}\right) = 4$
∴ $100 \left(\frac{X+5-X}{X(X+5)}\right) = 1$
∴ $100 (5) = X (X + 5)$
∴ $X^2 + 5X – 500 = 0$

(c) (i) Speed of Raj's car:
To get value of X, we need to solve the quadratic equation:
$X^2 + 5X – 500 = 0$
$X^2 + 25X – 20X – 500 = 0$
$(X + 25) (X – 20) = 0$
∴ X = – 25 and X = 20
Since value of speed can not be negative X ≠ – 25; Therefore, X = 20 km/h

(d) Time taken by Ajay to travel 400 km:
Given that the speed of Ajay's car = X + 5
∵ The speed of Raj's car, X = 20 km/h
∴ The speed of Ajay's car = X + 5 = 20 + 5 = 25 km/h
∴ Time taken to travel 400 km by Ajay's car = $\frac{400}{25}$ = 16 hrs

27. . (a) Let breadth of x m
length = (x + 1.5) m
∴ Area of plywood = $3m^2$
Quadratic equation is
$(x + 1.5)x = 3$
$\Rightarrow x^2 + 1.5x = 3$

$\Rightarrow x^2 + 1.5x - 3 = 0$

(b) We have $x^2 + 1.5x - 3 = 0$ $\Rightarrow 2x^2 + 3x - 6 = 0$

$x = \frac{-3 \pm \sqrt{9+48}}{4}$

$X = \frac{\sqrt{57}-3}{4} = \frac{7.54-3}{4} = \frac{4.54}{4} = 1.13$ m

(c) As $x^2 - 3 = 0$

$\Rightarrow (x - \sqrt{3})(x + \sqrt{3}) = 0$

$\Rightarrow x = \sqrt{3} = 1.73$ m

28. (a) (iii) (b) (ii) (c) (iii)

29. (a) (iii) (b) (ii) (c) (iii)

30. (i) Given, Hockey Rs. x per student and Cricket Rs. y per student

Algebraic equations are

$5x + 4y = 9500$ …… (1)

$4x + 3y = 7370$ ……... (2)

(ii) (a) Multiply by 3 in equation (1) and by 4 in equation (2)

$15x + 12y = 28,500$ ……... (3)

$16x + 12y = 29480$ ……. (4)

On subtracting equation (3) from equation (4),

we get $x = 980$

Prize amount for hockey = ₹980 **(OR)**

(b) Now, put this value in equation (1), we get

$5 \times 980 + 4y = 9500$

$4y = 9500 - 4900 = 4600$

$y = 1150$

Prize amount for cricket = Rs. 1150

Difference = 1150-980 = Rs. 170

Prize amount for cricket is Rs. 170 more than hockey.

(iii) Total prize amount for 2 students each from two games

$= 2x + 2y$

$= 2(x + y)$

$= 2(980 + 1150) = 2 \times 2130$

= Rs. 4260

CHAPTER 5-ARITHMETIC PROGRESSIONS

MULTIPLE CHOICE QUESTIONS

1) The flower arrangement of Raman sadan follows an AP in the order 10,6,2,…… . then, the sum of the 16 terms is …….. (AO2)
   (a) -320 (b) 320 (c)-352 (d)-400

2) PT madam arranged the students of class 7 in an order, so that it followed an AP. Find the 10th term of the A.P. 4, 9,14, …, 254 (AO1)
   (a) 41 (b) 44 (c) 46 (d) 49

3) The sum of first ten natural number is ……. (AO1)
   (a) 55 (b)155 (c) 65 (d) 110

4) The value of x for which $2x$, $x + 10$, and $3x + 2$ are the three consecutive terms of an AP (AO2)
   (a) -6 (b)18 (c)6 (d)-18

5) In an AP, the common difference is -4, the seventh term is 4, and then find the first term? (AO2)
   (a) 20 (b)28 (c)24 (d)21

6) The nth term of an A.P. is given by $a_n = 3 + 4n$. The common difference is (AO2)
   (a) 7 (b) 3 (c) 4 (d) 1

7) If the common difference of an AP is 3, then $a_{20} - a_{15}$ is (AO2)
   (a) 5 (b) 3 (c) 15 (d) 20

8) The first and last term of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be: (AO2)
   (a)5 (b)6 (c) 7 (d)8

ASSERTION AND REASONING (1 MARKS EACH)

In the following question a statement of Assertion (A) is followed by a statement of reason (R), choose the correct answer out of the following cases:

(a) Both A and R are true and R is the correct reason of A.
(b) Both A and R are true and R is not the correct reason of A.
(c) A is true but R is false.
(d) A is false but R is true.

1) **ASSERTION** (A): Sum of natural number from 1 to 100 is 5050 (AO1)
   REASON (R): The formula of general term an is $a_n = a + (n- 1)d]$

2) **ASSERTION**(A): The value of n, if a =10 , d= 5, a_n =95 is 18. (AO2)

   REASON (R): The formula of general term an is a_n=2n **[2a + (n − 1)d]**

3) ASSERTION (A) : The sum of the series with the nth term an= 9 – 5n is 220 when number of terms n = 6

REASON (R) : Sum of first n terms in an AP is given by the formula S_n= n/2 [2a + (n− 1)d] (AO1)

4) ASSERTION (A) : If numbers a, b, c are in AP then b - a = c – b (AO2)

REASON(R): Given three numbers are in AP then the common difference will be same

2 MARK QUESTIONS

1) Write first four terms of the AP,when first term is 1.25 and common difference is -0.25 (AO1)

2) Find the sum of all natural numbers from 1 to 100. (A01)

3) Find the missing terms in the given AP 2, ---, 26, --- (AO2)

4) Find the common difference of an AP in which a18 – a14 = 32. (AO2)

5)The first three terms of the AP are 3y-1,3y+5 and 5y+1 respectively. Find the value of y (AO2)

3 MARK QUESTIONS

1) If the sum of first m terms of an AP is am^2 + bm, find the common difference. (AO2)

2) The arrangement of flower pots in a garden followed an AP in the order 20, 17, 14, ...;

What is the first negative term? (AO2)

3) If the sum of first n terms of an AP is n^2 find the 5th term (AO2)

4) Find the sum of all odd numbers between 10 and 200. (AO1)

5) If the first three terms of an AP are b,c and 2b , find the ratio of b and c (AO2)

5 MARK QUESTIONS

1) If m^{th} term of an AP is 1 n and nth term is 1 Show that $(mn)^{th}$ term of this AP is 1.

2) If the 10th term of an A.P. is 52 and the 17th term is 20 more than the 13th term, find A.P

3) The 14th term of an A.P. is twice its 8th term. If the 6th term is -8, then find the sum of its first 20 terms.

CASE BASED QUESTIONS

CASE STUDY I

Raji decided to arrange the flower pots in her balcony in a particular order . She bought few plants with pots for her balcony. She placed the pots in such a way that the number of pots in the first row is 2, second row is 5, third row is 8 and so on

Based on the above information, answer the following questions:

i. Find the number of pots placed in the 10th row. (AO1)

ii. Find the difference in the number of pots placed in 5th row and 2nd row. (AO2)

iii. If Raji wants to place 101 pots in total, then the find total no of rows formed in the arrangement. (AO2)

CASE STUDY 2

Sahil takes a loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month , answer the following

i. . Find the amount paid by him in 30th instalment. (AO1)

ii. Find the total amount paid by him after 30 instalments. (AO1)

iii)How much money he had to pay after paying 30 instalments (AO2)

CASE STUDY 3

The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

i) 1. Find the production during first year. (AO2)

ii) Find the production during first 3 years. (AO2)

iii) Find the difference of the production during 7th year and 4th year. (AO1)

CASE STUDY 4

Rasmi decided to spend her pocket money which her parents gave her . She started buying two of her favourite chocolates every day after school and her savings started to decrease . the pattern of decrease followed in this manner 27,24,21,…..0

i) What was her saving on the fifth day? (AO1)

ii) What was the original saving and by what amount it was decreasing? (AO1)

iii) On which day her savings became 0? (AO2)

CASE STUDY 5

Seeta arranged fruit loops on a table . She realised that difference between successive groups of fruit loops was the same and identified it as a pattern. Hence it formed an AP. The AP was identified as 5,9,13……

i) How many fruit loops were there in the tenth group.? (AO1)

ii) What is the difference in the number of loops in 7th group and 9th group? (AO2)

iii) Find the number of fruit loop groups if the last group has 185 loops? (AO2)

SOLUTIONS: CHAPTER 5-ARITHMETIC PROGRESSIONS

MULTIPLE CHOICE QUESTIONS

1)-320

2)49

3)55

4)6

5)28

6)4

7)15

8)6

ASSERTION ANHD REASON QYESTIONS

1) b

2)c

3)d

4)a

2 MARK QUESTIONS

1) a, a+d, a+2d, a+3d= 1.25, 1 ,0.75, 0.50

2) $n(n+1)$ 2 = 100(100+1) 2 = 5050

3) a2= a1+a3= $\frac{2+26}{2}$=14,

d= a_2 - a_1=12

a_4= a_3 + d = 26+12=38

4) $a_{18} - a_{14}$ = 32. a + 17d - (a + 13d) = 32,

4d=32,

d=32÷4

=8

5) 3y+5= $\frac{3y-1+5y+1)}{2}$

y =5

3 MARK QUESTIONS

1) $S_m = am^2 + bm$, $S_1 = a + b = a_1$,

$S_2 = 4a+2b = a_1 + a2$

$a_2 = S_2 - S_1 = 4a+2b - (a + b) = 3a + b$,

$d = a_2 - a_1 = 3a+b - (a + b) = 2a$

2) 8th term is the first negative number

3) Given $Sn = n^2$,

we know , $an = Sn - S(n - 1)$,

$a5 = S5 - S(5 - 1) = S5 – S4 = 52-42 = 25-16=9$

4) Odd numbers between 10 and 200 are 11,13,15....199.

$a1 = 11$,

Last term $l = 199$, $d = 2$,

$an = a + (n-1) d$,

$199 = 11 + (n - 1) 2$,

$199 - 11 = (n - 1) 2$,

$188 = (n - 1) 2$,

$94 = n – 1$

$,95 = n$

Sum of n terms = n 2 (a + l), =95 2 (11 + 199)= 9975

5) $c = b+2b/2 = 3b/2$

$2c = 3b$

$b/c = 2/3$

$b:c=2:3$

5MARK QUESTIONS

1) mth term $=\frac{1}{n}$ nth term$=\frac{1}{m}$

$a+(m-1)d=\frac{1}{n}$ $a+(n-1)d= \frac{1}{m}$

on simplifying $d=\frac{1}{mn}$, $a= \frac{1}{mn}$

mnth term= $a+(mn-1)d = \frac{1}{mn}+(mn-1) \frac{1}{mn}=1$

2) Given,

$a_{10} = 52 \Rightarrow a+9d=52$ .....(1)

also,

$a_{17} = 20 + a_{13} \Rightarrow a + 16d = 20 + a + 12d \Rightarrow 16d - 12d = 20 \Rightarrow 4d = 20 \Rightarrow d=5$
putting the value of d in eq. (1),
we get, $\Rightarrow a+9(5)=52 \Rightarrow a+45=52 \Rightarrow a=7$
hence, the required AP is 7,12,17,.....
3) Let the first term is a and common difference is d
Here, $a_{14} = 2\ a\ 8$
Or, a+ 13 d = 2(a+7d) a + 13d
= 2a + 14d -a -13d = 0,
a = - d ……(1) again
a_6 = - 8 or a + 5d = - 8 ……..(2)
solving eq. (1) and (2) we get a = 2 , d = -2
$S_{20} = 10\ (4 + (-38)) = 10\ (\ 4 – 38\ ) = -340$

CASE BASED QUESTIONS

CASE STUDY-1

a)29

b) 9

c) 34

CASE STUDY- 2

a)3900

b)73500

c)44500

CASE STUDY- 3

a)5000

b)21600

c)6600

CASE STUDY -4

a)15

b) 27 and 3

c)n=10

CASE STUDY -5

a)41

b)8

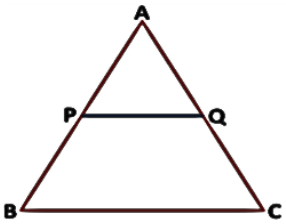
c)46

CHAPTER 6-TRIANGLES

MULTIPLE CHOICE QUESTIONS

1) In the given figure PQIIBC, If AP=3cm, PB= 4 cm and AQ= 6 cm find QC (AO1)

a)2cm b) 6cm c) 8cm d)10 cm



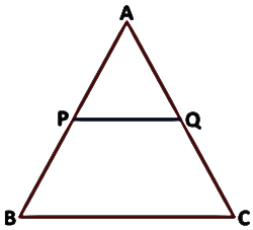
2) In triangles ABC and DEF, ∠B = ∠E, ∠F = ∠C and AB = 3DE. Then, the two triangles are (AO1)

a) congruent but not similar b) similar but not congruent

c) neither congruent nor similar c) congruent as well as similar

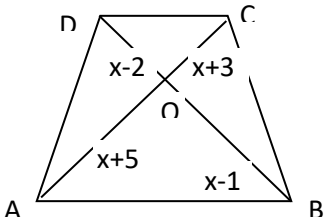
3) In the given figure in triangle ABC, PQ II BC and $\frac{AP}{PB} = \frac{3}{5}$ AQ= 28cm, find QC (AO1)

a) 10cm b) 10.5 cm c) 46.6 cm. d)15 cm



4) In the given figure, if AB II DC, find the value of x (AO2)

a) 8cm b) 7cm c) 6 cm d) 4cm



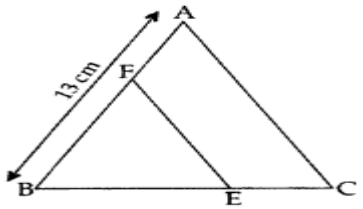
5) If in two Δs ABC and DEF, $\frac{AB}{DF} = \frac{BC}{EF} = \frac{CA}{ED}$, then (AO2)

a) ΔABC ~ ΔDEF b) ΔABC ~ ΔEDF c) ΔABC ~ ΔEFD d) ΔABC ~ ΔDFE

6) In ΔABC, DE II AC, AE = a units, EC = b units DE = x units BC=y units (AO2)

a) x=$\frac{a+b}{ay}$ b) y=$\frac{ax}{a+b}$ c) x = $\frac{ay}{a+b}$ d) $\frac{x}{y} = \frac{a}{b}$

7) In the figure, EF || AC, BC = 10 cm, AB = 13 cm and EC = 2 cm, find AF. (AO2)



a) 3.4cm b)2.6 cm c)2cm d)2.5cm

8) In ΔLMN, ∠L = 50° and ∠N = 60°, If ΔLMN ~ ΔPQR, then find ∠Q (AO2)

a) 40° b) 50° c) 70° d)120°

ASSERTION AND REASON QUESTIONS

In the following question a statement of Assertion (A) is followed by a statement of reason (R), choose the correct answer out of the following cases:

(a) Both A and R are true and R is the correct reason of A.
(b) Both A and R are true and R is not the correct reason of A.
(c) A is true but R is false.
(d) A is false but R is true.

1) **A: ASSERTION**: A line drawn parallel to any one side of a triangle intersects the other two sides proportionally.

R: REASON: Parallel lines cannot be drawn to any one side of a triangle. (AO1)

2) **A: ASSERTION**: If two angles of any triangle are equal to the corresponding two angles of another triangle then the third angles are not necessarily equal. (AO2)

R: REASON: The sum of three angles of any triangle is equal to 180°

3) **A: ASSERTION**: If ΔABC *and* PQR are congruent then they are also similar triangles

R: REASON: All congruent triangles are similar but all similar triangles need not be congruent (AO2)

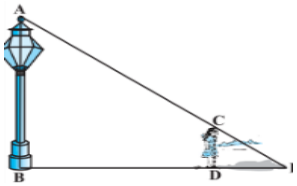
4). E and F are the points on the sides PQ and PR respectively of a triangle PQR. PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm**.**

A: ASSERTION: EF is not parallel to QR

R: **REASON**: In a triangle if two sides are divided proportionally by a line then the line is parallel to the third side. (AO1)

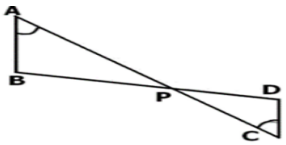
2MARK QUESTIONS

1) A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5m casts a shadow of 3m, what is the length of the shadow of the pole? (AO2)

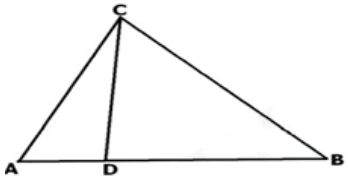


2) If In the below figure, if ∠A = ∠C, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD

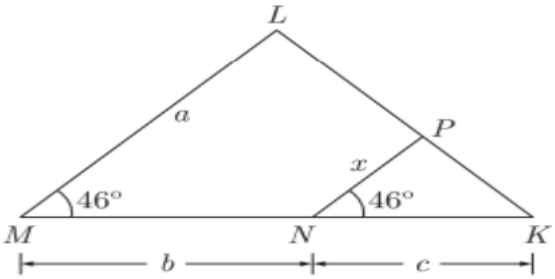
(AO2)



3) In the figure, if ∠ACB= ∠ CDA, AC=8 cm and AD = 3 cm, find BD. (AO2**)**



4) In the figure find x (AO2)



5) In ΔABC, if DE || BC, AD = x, DB = x – 2, AE = x + 2 and EC = x – 1, then value of x is (AO2)

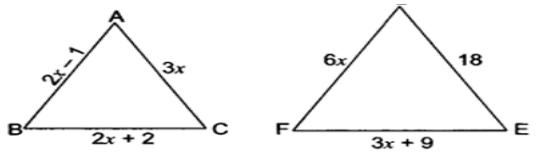
3 MARKS QUESTIONS

1) In ΔABC, D and E are points on sides AB and AC respectively such that DE II BC and

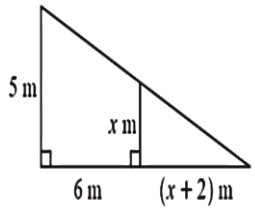
AD: DB = 3:1 If EA = 6.6cm , find AC (AO2)

2 In fig, if ΔABC ~ ΔDEF and their sides are of lengths (in cm) as marked along with them, then find

the lengths of the sides of each triangle. (AO2)



3) Find the value of x (AO2)



4) In the ΔABC, DE II BC and AD=$\frac{1}{2}$ BD. If BC= 4.5 cm , find DE. (AO2)

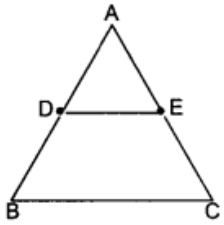
5) In ΔABC, DE‖ BC such that AD = 7x - 4 cm, AE = 5x - 2 cm , DB = 3x + 4 cm and EC = 3x cm .

then find the value of x. (AO2)

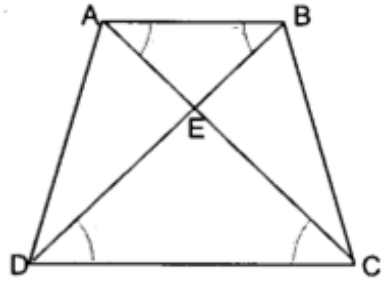
5 MARK QUESTIONS

1) a) Prove that, if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. (AO1)
Using the above result, do the following:

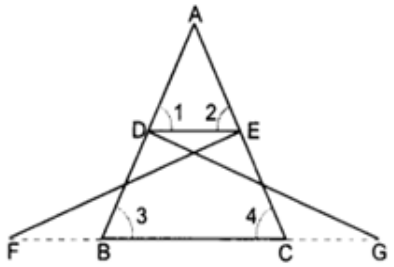
b) In Fig. DE || BC and BD = CE. Prove that ΔABC is an isosceles triangle. (AO2)



2) ABCD is a trapezium with AB || DC. If ΔAED is similar to ΔBEC, prove that AD = BC. (AO2)



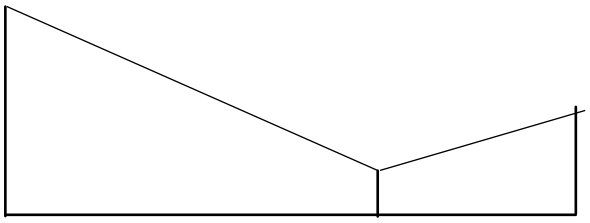
3) Δ FEC ≅ Δ GDB and ∠1 = ∠2. Prove that ΔADE ~ ΔABC (AO2)



CASE BASED QUESTIONS

CASE STUDY- 1

Rani places a mirror on level ground to determine the height of a pole (with traffic light fired on it). She stands at a certain distance so that she can see the top of the pole reflected from the mirror. Rani's eye level is 1.5 m above the ground. The distance of Rani and the pole from the mirror are 1.8 m and 6 m respectively.



a) Which criterion of similarity is applicable to similar triangles? (AO1)

b) What is the height of the pole? (AO2)

c) What is the distance between mirror and pole? (AO2)

CASE STUDY- 2

Rajni and Sajini were standing in the ground in front of a light house in mamallapuram:. Rajni observed that the tower casts a shadow of 50m long and Height of Rajnis house is 20m and it casts a shadow of 10m on the ground. If Sajni 's house casts a shadow 20m on the **ground.**

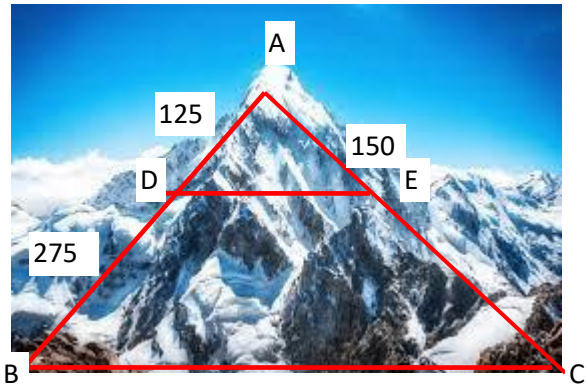
a) What is the height of the tower? (AO2)

b) What is the height of Sajini's house? (AO2)

c) What is the similarity criteria used? (AO1)

CASE STUDY- 3

Mountaineering is the perfect activity for adventure lovers. Every year, several mountaineers attempt to climb the Mount Everest.



The path of two mountaineers from the base camps B and C are shown above. D and E are two mid camping areas in between their paths and the line joining D and E is parallel to the line joining B and C.

1) Find the distance between E and C (AO1)
2) What is the ratio of the distance between DE and BC? (AO2)
3) What is the theorem used here? (AO1)

CASE STUDY- 4

Sanket wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 PM, he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 12.5 feet.

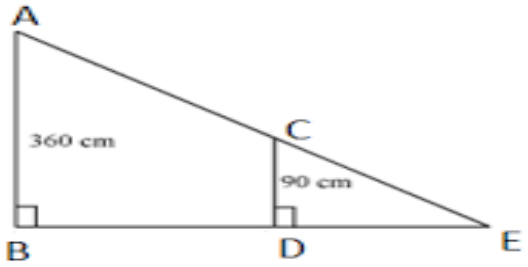
a) What is the height of the tree? (AO1)

b) Find the ratio of heights of tree and fence? (AO2)

c) Find the similarity criteria used? (AO1)

CASE STUDY - 5

A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.



a) Find the length of her shadow after 4 seconds. (AO2)

b)What is the similarity criteria used? (AO1)

c)Find the distance BE? (AO2)

SOLUTIONS: CHAPTER 6-TRIANGLES

MULTIPLE CHOICE QUESTIONS

1)8cm

2)similar but not congruent

3)46.6

4)7cm

5) ΔABC ~ ΔDFE

6)) x=$\frac{ay}{a+b}$

7) 2.6cm

8) 70°

ASSERTIONM AND REASON QUESTIONS

1)iii

2)iv

3)i

4)iv

2 MARK QUESTIONS

1) Let AB be the pole and PQ be the height of the woman.

∠ABC=∠PQR=90, ∠ACB=∠PRS

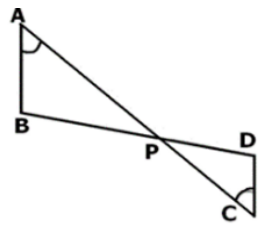
ΔABC ~ ΔPQR (AA) similarity

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\frac{6}{1.5} = \frac{BC}{3}$

Length of shadow of the pole= BC=12 m.

2) $\frac{AB}{CD} = \frac{AP}{CP} = \frac{BP}{PD}$



CD=2 cm, PD= 5 cm.

3) Δ ADC ~ ΔACB (AA similarity)

$\frac{AC}{AD} = \frac{AB}{AC}$

$\frac{8}{3} = \frac{AB}{8}$

AB= 64/3 cm

BD= $\frac{64}{3}$ -3 =55/3 cm

4) ΔKNP ~ ΔKML , Using AA similarity $\frac{KN}{KM} = \frac{PN}{LM}$

Then, $\frac{c}{b+c} = \frac{x}{a}$.

That is, $x = \frac{ac}{b+c}$

5) In ΔABC, we have DE || BC, $\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem] $\Rightarrow \frac{x+2}{x-1} = \frac{x}{x-2}$

x = 4

3 MARK QUESTIONS

1) $\frac{AD}{DB} = \frac{3}{1} = \frac{AE}{EC}$

EC x 3=6.6

EC= 6.6/3= 2.2 cm

AC= 8.8cm

2) $\Delta ABC \sim \Delta DEF$(Given)

$\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$

$\frac{2x-1}{18}=\frac{2x+2}{3x+9}=\frac{3x}{6x}$

$\frac{2x-1}{18}=\frac{3x}{6x}$ $\frac{2x-1}{18}=\frac{1}{2}$, x = 5

AB = 9 , BC = 12, CA = 15, DE = 18 ,EF = 24, FD = 30

3) $\frac{AD}{DB}=\frac{AE}{EC}$.

$\frac{5}{x}=\frac{x+8}{x+2}$

5(x+2)= x(x+8)

5x+10= x^2+8x $x^2+3x-10$

X=2,-5

4) AD=$\frac{1}{2}$ BD. $\frac{AD}{BD}=\frac{1}{2}$

$\frac{BD}{AD}$=2

$\frac{BD}{AD}$+1= 2 +1

$\frac{AB}{AD}$ = 3

2AD=BD

BC= 4.5 cm

$\frac{AD}{AB}=\frac{DE}{BC}$

$\frac{1}{3}=\frac{DE}{4.5}$

DE= $\frac{4.5}{3}$= =1.5cm

5)

$\frac{7x-4}{3x+4}=\frac{5x-2}{3x}$

$3x^2-13x+4=0$

x=4,1/3

If x=1/3, 7x-4=-5/3<0, not possible.

Therefore, x=4

5 MARK QUESTIONS

1)

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$.

Construction: Join BE and CD and then draw $DM \perp AC$ and $EN \perp AB$.

Proof: Area of ΔADE $= \left(\frac{1}{2} \text{ base} \times \text{height}\right)$.

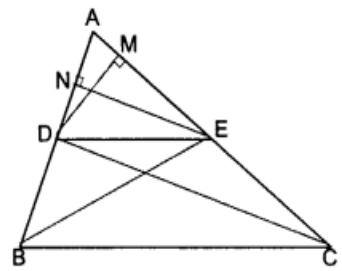


Fig. 7.45

So, $ar(\Delta ADE) = \frac{1}{2}(AD \times EN)$

and $ar(\Delta BDE) = \frac{1}{2}(DB \times EN)$

Similarly, $ar(\Delta ADE) = \frac{1}{2}(AE \times DM)$

and $ar(\Delta DEC) = \frac{1}{2}(EC \times DM)$

Therefore, $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ...(*i*)

and $\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$ ...(*ii*)

Now, ΔBDE and ΔDEC are on the same base DE and between the same parallel lines BC and DE.

So, $ar(\Delta BDE) = ar(\Delta DEC)$ ...(*iii*)

Therefore, from (*i*), (*ii*) and (*iii*) we have, $\frac{AD}{DB} = \frac{AE}{EC}$

Second Part

As $DE \| BC$

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \Rightarrow \quad \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\Rightarrow \quad \frac{AD + DB}{DB} = \frac{AE + EC}{EC} \quad \Rightarrow \quad \frac{AB}{DB} = \frac{AC}{EC}$

Solution:

Given: A triangle ABC in which a line parallel to side BC intersects other two sides

⇒ AB = AC (As DB = EC)

∴ΔABC is an isosceles triangle

2) Solution:

In ΔEDC and ΔEBA we have

∠1 = ∠2 [Alternate angles]

∠3 = ∠4 [Alternate angles]

∠CED = ∠AEB [Vertically opposite angles]

∴ΔEDC ~ ΔEBA [By AA criterion of similarity]

$$\Rightarrow \quad \frac{ED}{EB} = \frac{EC}{EA} \quad \Rightarrow \quad \frac{ED}{EC} = \frac{EB}{EA} \qquad \ldots(i)$$

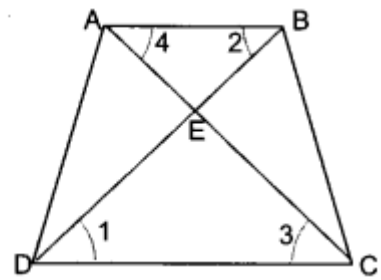


Fig. 7.37

It is given that $\Delta AED \sim \Delta BEC$

$$\therefore \quad \frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC} \qquad \ldots(ii)$$

From (i) and (ii), we get

$$\frac{EB}{EA} = \frac{EA}{EB} \quad \Rightarrow \quad (EB)^2 = (EA)^2 \quad \Rightarrow \quad EB = EA$$

Substituting $EB = EA$ in (ii), we get

$$\frac{EA}{EA} = \frac{AD}{BC} \quad \Rightarrow \quad \frac{AD}{BC} = 1 \quad \Rightarrow \quad AD = BC$$

3)

$$\Rightarrow \quad EC = BD \qquad \ldots(i)$$

It is given that

$$\angle 1 = \angle 2$$

$$\Rightarrow \quad AE = AD \qquad \left[\begin{array}{l}\text{Sides opposite to equal}\\ \text{angles are equal}\end{array}\right] \qquad \ldots(ii)$$

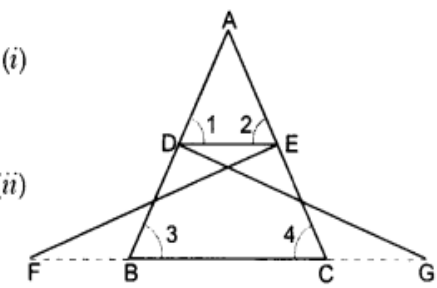


Fig. 7.57

From (i) and (ii), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$\Rightarrow \quad DE \parallel BC$ [By the converse of basic proportionality theorem]

$\Rightarrow \quad \angle 1 = \angle 3$ and $\angle 2 = \angle 4$ [Corresponding angles]

Thus, in Δ's ADE and ABC, we have

$\angle A = \angle A$ [Common]

$\angle 1 = \angle 3$

$\angle 2 = \angle 4$ [Proved above]

So, by AAA criterion of similarity, we have

$$\Delta ADE \sim \Delta ABC$$

CASE BASED QUESTIONS

CASE STUDY- 1

a) AA similarity
b) h = 5m
c) 7.8m

CASE STUDY- 2

a) 100m
b) 40 m
c) AA similarity

CASE STUDY- 3

a) 330 km
b) 5:11
c) BPT

CASE STUDY- 4

a) 20
b) 20:4=5:1
c) AA similarity

CASE STUDY - 5

a) x = 1.6 m
b) AA similarity
c) 6.4 m

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CHAPTER 7-COORDINATE GEOMETRY

MULTIPLE CHOICE QUESTIONS

1. A new pedestrian walkway between two landmarks R(−1,3) and S(9,8) in a city. A rest area is to be placed along the walkway at point P, which divides the walkway in the ratio k:1. Additionally, the rest area must lie along an existing bike path defined by the equation x−y+2=0. Determine the value of k that will ensure the rest area is correctly positioned. (A02)

   a) $\frac{2}{3}$ b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

2. A town has a rectangular park with opposite corners at coordinates (1, 3) and (5, 6). What is the length of the diagonal path of the park? (A01)

   a. 5 units b. 6 units c. √32 units d. 7 units

3. A drone flies from point A(2,3) to point B(5,7).What is the distance covered by the drone? (A01)

   a. 5 units b. √18 units c. √20 units d. 6 units

4. In a city, there are two street lights at positions (2,3) and (5,7). A tree is to be planted at the midpoint of the line segment connecting these two streetlights. Find the coordinate of the tree. (A01)

   a. (3, 5) b. (3.5, 5) c. (4, 5) d. (3.5, 6)

5. A rectangular playground has its vertices at (0, 0), (0, 6), (8, 0), and (8, 6). What is the area of the playground? (A02)

   a. 24 sq. units b. 36 sq. units c. 48 sq. units d. 64 sq. units

6. A farmer wants to divide his triangular field into two equal areas by drawing a line. The vertices of the triangle are (0, 0), (6, 0) and (3, 6).What will be the length of the line through (3, 6)?(A02)

   a. 5 units b. 4 units c. 6 units d. 3 units

7. What type of triangle can you form in a garden with three flower beds located at points (−4,0), (4,0), and (0,3). (A02)

   a) right triangle b) isosceles triangle c)equilateral triangle d) scalene

8. A town has a rectangular park with opposite corners at coordinates (1, 2) and (5, 6). What is the length and breadth of the park? (A02)

   a. 4 units,4 units b. 6 units,8 units c. 5 units, 3 units d. 3 units ,11 units

ASSERTION REASON QUESTIONS

DIRECTION: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of Assertion (A).
b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of Assertion (A).
c) Assertion (A) is true but reason (R) is false.
d) Assertion (A) is false but reason (R) is true.

1. Assertion (A) : The point P(- 4, 6) divides the join of A (- 6, 10) and B(3 - 8) in the ratio 2 :7
   Reason (R) : If the point P(x, y) **divides** the line segment joining A(x_1, y_1) and B(x_2, y_2) **internally** in the **ratio m:n**, then, the coordinates of P are given by
   $$P(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$ **(A01)**

2. Assertion (A): Distance of the point (4,7) from the y axis is 4 units. **(A01)**
   Reason(R) : Distance of a point from y axis is given by its x coordinate

3. Assertion (A) : The distance between the points (cos Θ, sin Θ) and (sin Θ, -cos Θ) is 2 units. **(A02)**
   Reason (R) : The distance between A (x_1,y_1) and B (x_2,y_2) is given by
   $$AB = \sqrt{[x_2 - x_1]^2 + [y_2 - y_1]^2}$$

4. Assertion (A): If the co-ordinates of the mid-points of the sides AB and AC of Δ ABC are D(3,5) and E(-3,-3) respectively, then BC = 20 units.
   Reason (R): The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it. **(A02)**

2 MARKS QUESTIONS

1. As an urban planner designing a new park, you have a central fountain located at the coordinates (3,4). You need to place a statue in the park such that: (A02)
   a. The statue is exactly $\sqrt{10}$ units away from the fountain.
   b. The x-coordinate of the statue is twice its y-coordinate.

   What are the possible coordinates of the statue that satisfy these conditions?

2. You are planning to place benches $2\sqrt{5}$ units away from a notable tree located at the coordinates (7,−4) on a park layout. If all these lie on a straight line, how many possible positions can you place the benches along this pathway to satisfy the distance requirement from the tree? Find their coordinates. (A02)

3. **Landmark A** is located at coordinates (4,5)and **Landmark B** is at coordinates (−2,3). Find the equation of a new walkway that is the perpendicular bisector of the straight path connecting these two landmarks. (A02)

4. A football field is in the shape of a parallelogram with vertices at (1,2), (4,6), (7,2) and (10,6). Calculate the area of the football field. (A02)

5. The bottom-left corner of the rectangular garden is at the coordinates (2,3) and the top-right corner is at the coordinates (8,7). What are the coordinates of the top-left and bottom-right corners? (A01)

3 MARKS QUESTIONS

1. A surveillance camera needs to be placed such that it is equidistant from three key points in a park: a fountain at (2,3) a statue at (8,5) and a bench at (4,7). What are the coordinates of the point where the camera should be placed? (A02)

2. A park is to be designed in the shape of a trapezoid with vertices at A(2,3), B(8,3) C(5,7) and D(1,7). Calculate the length of the mid-segment (the segment that connects the midpoints of the non-parallel sides). (A02)

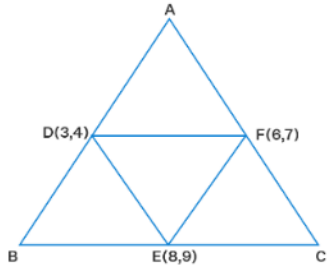
3. A triangular plot of land has vertices at A(2,3), B(5,7) and C(9,3). Find the length of each side of the triangle and classify the triangle by its sides. (A02)

4. John and Mary are planning to meet for lunch. John is at his office located at point A (2, 3) and Mary is at her home located at point B (10, 7). They decide to meet at a café that is exactly two-fifths of the way from John's office to Mary's home. Find the coordinates of the café. (A01)

5. A straight road connects a school located at point A (1, 2) and a hospital located at point B (11, 8). A grocery store is located at point C (6, 5) along this road. Determine the ratio in which the grocery store divides the road between the school and the hospital. (A01)

5 MARKS QUESTIONS

1. Archaeologist have discovered three significant markers that are believed to be the midpoints of the sides of a large triangular foundation. These markers are at coordinates D(3,4), E(8,9) and F(6,7). Determine the original vertices of this triangular foundation to understand the full layout of the ancient structure. **(A02)**

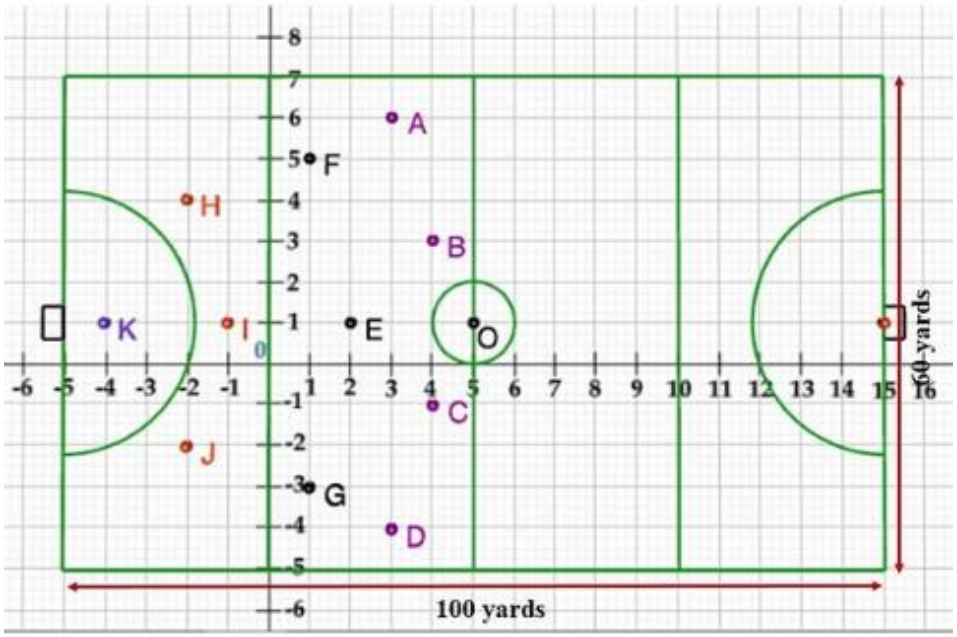


2. Three of the posts have already been placed in a garden at the following coordinates:Post A at (-9, -2) ,Post B at (4, -2) , Post C at (6, 5)
You need to determine where to place the fourth post (D) to complete the parallelogram-shaped garden. Additionally, you need to calculate the area of the garden to determine the amount of grass seed needed to cover it. **(A02)**

3. The base *QR* of an equilateral triangle *PQR* lies on *x*-axis. The co-ordinates of point *Q* are (-4, 0) and the origin is the mid-point of the base. Find the coordinates of the point *P* and *R*. **(A02)**

CASE BASED QUESTIONS

1. A hockey field is the playing surface for the game of hockey. Historically, the game was played on natural turf (grass) but nowadays it is predominantly played on an artificial turf. It is rectangular in shape - 100 yards by 60 yards. Goals consist of two upright post placed equidistant from the centre of the backline, joined at the top by a horizontal crossbar. The inner edges of the posts must be 3.66 metres (4 yards) apart, and the lower edge of the crossbar must be 2.14 metres (7 feet) above the ground.

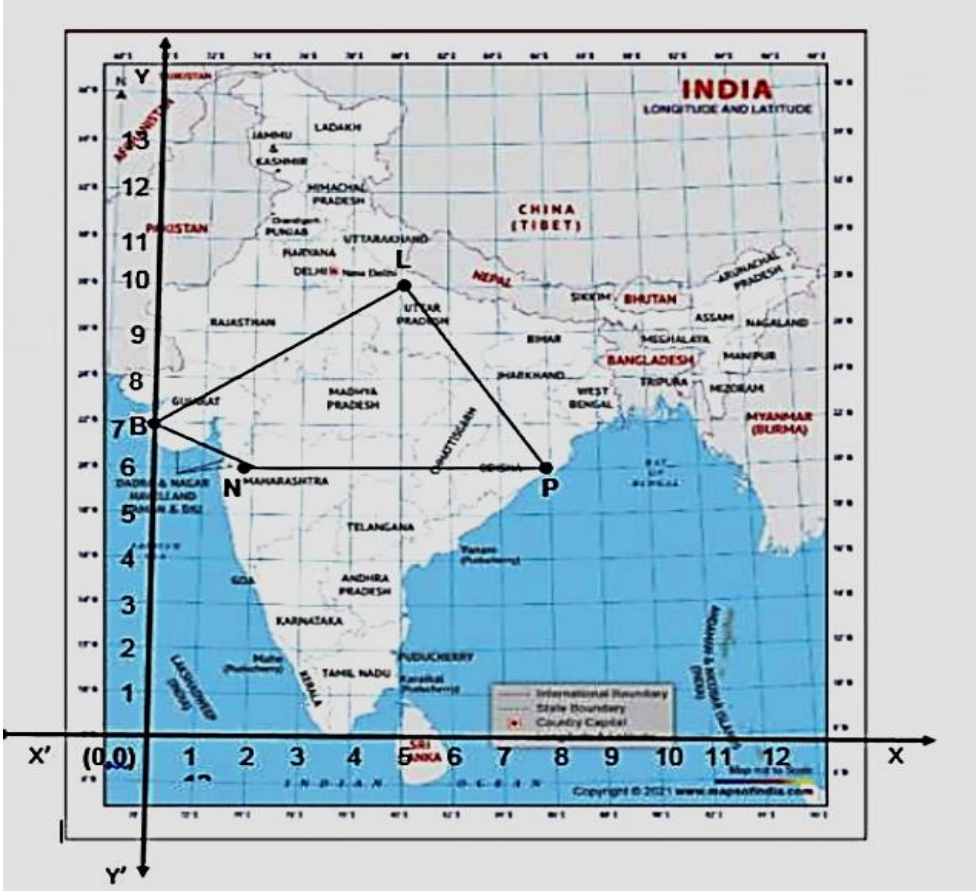


Each team plays with 11 players on the field during the game including the goalie.
Positions you might play include-
• Forward: As shown by players A, B, C and D.
• Midfielders: As shown by players E, F and G.
• Fullbacks: As shown by players H, I and J.
• Goalie: As shown by player K Using the picture of a hockey field below,answer the questions that follows

i. If a player P needs to be at equal distances from A and G, such that A, P and G are in straight line, then find position of P . **(A02)**

ii. What are the coordinates of the position of a player Q such that his distance from K is twice his distance from E and K, Q and E are collinear? **(A02)**

2. In a GPS, the lines that run east-west are known as lines of latitude, and the lines running north-south are known as lines of longitude. The latitude and the longitude of a place are its coordinates and the distance formula is used to find the distance between two places. The distance between two parallel lines is approximately 150 km. A family from Uttar Pradesh planned a round trip from Lucknow (L) to Puri (P) via Bhuj (B) and Nashik (N) as shown in the given figure below.



Based on the above information answer the following questions using co- ordinate geometry

i) Find the distance between Lucknow to Bhuj (A01)

ii) If Kota internally divides a line segment joining Lucknow to Bhuj in the ratio 3:2 then find the coordinates of Kota . (A01)

iii) Name the type of Triangle formed by the places Lucknow Nashik and Puri (A02)

3. Tharunya was thrilled to know that the football tournament 20th July to 20th August, 2023 and for the first time in the FIFA Women's World Cup's history ,two nations host in 10 venues. Her father felt that the game can be better understood if the position of two players is represented as points on a coordinate plane (A02)

i) At an instance, the midfielders and forward formed a parallelogram. Find the postion of the central midfielder (D) if the position of other players who formed the parallelogram are: A(1, 2), B(4,3) and C(6,6).

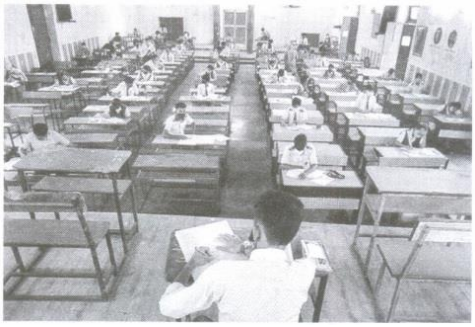
ii) Check if the Goalkeeper G(-3,5), Sweeper H(3,1) and Wing-back K(0,3) fall on a same straight line.

OR

Check if the full-back J(5,−3) and centre-back I(−4,6) are equidistant from forward C(0,1) and if C is the mid-point of IJ.

iii) If Defensive midfielder A(1, 4), Attacking midfielder B(2,−3) and Striker E(a,b) lie on the same straight line and B is equidistant from A and E, find the position of E.

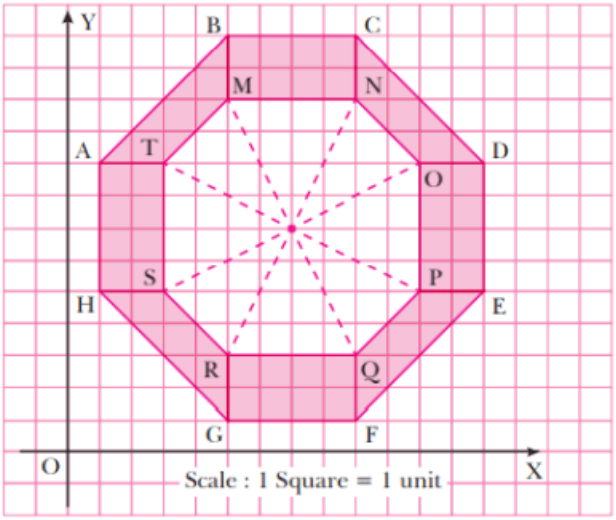
4. In an examination hall, students are seated at a distance of 2 m from each other, to maintain the social distance due to CORONA virus pandemic. Let three students sit at points A, Band C whose coordinates are (4, -3), (7,3) and (8, 5) respectively. (A02)



(i) If an invigilator at the point I, lying on the straight line joining Band C such that it divides the distance between them in the ratio of 1 : 2. Then find the coordinates of I.

(ii) Find the ratio in which B divides the line segment joining A and C.

5. The top of a table is shown in the figure given below: (A01)



i) Find the distance between the points A and B.

ii) Find the coordinates of the mid-point of line segment joining points M and Q

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SOLUTIONS: CHAPTER 7-COORDINATE GEOMETRY

MULTIPLE CHOICE QUESTIONS

1. a. $\frac{2}{3}$	2. a. 5 units	3. a. 5 units	4. b. (3.5, 5)
5. c. 48	6. c. 6 units	7. b. isosceles	8. a. 4 units,4 units

ASSERTION AND REASON QUESTIONS

1. a) 2. a) 3. d) 4. a)

2 MARKS QUESTIONS

1. .Let the coordinates of the point P be $(2x, x)$. Its distance from the point Q(3,4)is $\sqrt{10}$

So PQ^2 =10

$$(3-2x)^2+(4-x)^2=10$$

$$5x^2-20x+15=0$$

$$(x-3)(x-1)=0$$

$x=1\ or\ x=3$ Which gives the coordinates as P(6,3) or P(2,1)

2. Let the point P$(x, 0)$ of the X-axis be at a distance of $2\sqrt{5}$ units from the point A((7,−4) . Then

AP= $2\sqrt{5}$ AP$^2=20$

$$(x-7)^2+(0+4)^2=20$$

$(x-7)^2$=20-16= 4

$$(x-7) = \pm 2$$
$$x = 9 \; or \; x = 5$$

There are 2 required points which are (5,0) and (9,0)

3. Let the point P(x, y) lie on the perpendicular bisector of the line-segment joining the points A(4,5) and B$(-2,3)$

PA=PB, $PA^2 = PB^2$

$$(x-4)^2 + (y-5)^2 = (x+2)^2 + (y-3)^2$$
$$x^2 - 8x + 16 + y^2 - 10y + 25 = x^2 + 4x + 4 + y^2 - 6y + 9$$
$$-12x - 4y + 28 = 0$$
$$3x + y - 7 = 0$$

This is the equation of the perpendicular bisector of the line segment.AB

4. The area of a parallelogram is given by: A=base×height
   The base is the distance between (1,2)and 7,2), is 6 units
   The height is the vertical distance between the parallel sides, which can be found using the distance between (4,6)and(4,2): height=6−2=4 units
   A=6×4=24 square units

5. (2,7) and (8,3)

<u>3 MARKS QUESTIONS</u>

1. For the first pair of distances. (equidistant formula)
   $(x-2)^2 + (y-3)^2 = (x-8)^2 + (y-5)^2$ ……(1)
   For the second pair of distances:
   $(x-8)^2 + (y-5)^2 = (x-4)^2 + (y-7)^2$…….(2)
   3x+y=19 …….(1)
   2x−y=6……(2)
   Solving : x=5 , y=4 (5,4).

2. To find the mid-segment, first find the midpoints of the non-parallel sides AC and BD.
   Midpoint of AC: $M_1 = \left(\frac{2+5}{2}, \frac{3+7}{2}\right) = (3.5,5)$
   Midpoint of BD: $M_2 = \left(\frac{8+1}{2}, \frac{3+7}{2}\right) = (4.5,5)$
   length of the mid-segment: $M_1M_2 = \sqrt{(4.5-3.5)^2 + (5-5)^2} = 1+0=1$
   So, the length of the mid-segment is 1 unit.

3. Length AB= 5 units
   BC = $4\sqrt{2}$ units.
   CA = 7 units
   Since all the sides have different lengths, the triangle is a scalene triangle.

4. To find the coordinates of the café that divides the line segment joining points A (2, 3) and B (10, 7) in the ratio 2:3, we use the section formula.

   Substituting the values:

$P\left(\frac{2\cdot10+3\cdot2}{2+3}, \frac{2\cdot7+3\cdot3}{2+3}\right) = (5.2, 4.6)$

So, the coordinates of the café are (5.2, 4.6).

5. $5 = \frac{11k+1}{k+1}$, $k = 2/3$

5 MARKS QUESTIONS

1. $[(x_1 + x_2)/2, (y_1 + y_2)/2] = (3, 4)$

$x_1 + x_2 = 6$ ---------- (1)

Also, $(y_1 + y_2)/2 = 4$

$y_1 + y_2 = 8$ ---------- (2)

$x_2 + x_3 = 16$……..(3)

$y_2 + y_3 = 18$ ---------- (4)

$x_1 + x_3 = 12$ ---------- (5)

$y_1 + y_3 = 14$ ---------- (6)

Adding (1), (3) and (5) we get,

$2(x_1 + x_2 + x_3) = 6 + 16 + 12$

$x_1 + x_2 + x_3 = 17$ ---------- (7)

Substitute (1) in (7),

$6 + x_3 = 17, x_3 = 11$

Substitute (2) in (7),

$x_1 + 16 = 17, x_1 = 1$

Substitute (3) in (7),

$x_2 + 12 = 17, x_2 = 5$

Adding (2), (4) and (6) we get,

$2(y_1 + y_2 + y_3) = 8 + 18 + 14$

$y_1 + y_2 + y_3 = 20$ ---------- (8)

Substitute (2) in (8),

$8 + y_3 = 20, y_3 = 12$

Substitute (4) in (8),

$y_1 + 18 = 20$

$y_1 = 2$

Substitute (6) in (8),

$y_2 + 14 = 20$

$y_2 = 6$

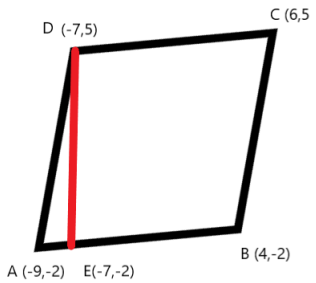
Therefore, the vertices of Δ ABC are A(1, 2) B(5, 6) and C(11, 12)

2. In a parallelogram, the diagonals bisect each other. Therefore, the midpoint of the diagonal AC should be the same as the midpoint of the diagonal BD

Midpoint of AC=$(\frac{-9+6}{2}, \frac{-2+5}{2}) = (\frac{-3}{2}, \frac{3}{2})$

B=(4,−2) , D = (x,y)

Midpoint of BD = $(\frac{4+x}{2}, \frac{-2+y}{2}) = (\frac{-3}{2}, \frac{3}{2})$



Solving, the coordinates of D are (−7,5).

Height = distance DE = 7

Area = ½ x AB x DE = ½ x 13 x 7 = 45.5 sq.units.

3. Since the origin (0, 0) is the midpoint of the base QR, the coordinates of point R can be determined using the midpoint formula

Therefore, the coordinates of point R are (4,0).

The height of an equilateral triangle of side 'a' is given by $\frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2}QR = \frac{\sqrt{3}}{2}8 = 4\sqrt{3}$.

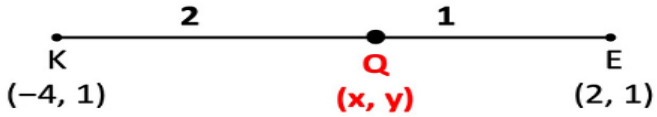
Thus, the coordinates of points P and R are $(0,4\sqrt{3}.)$and (4,0) respectively.

CASE BASED QUESTIONS

1. i) If a player P needs to be at equal distances from A and G, such that A, P, and G are in a straight line, then P must be the midpoint of the segment AG.

P= (2,3/2)

iii) QK=2 QE, QK: QE = 2: 1



Q = (0,1)

2. i) $150\sqrt{34}$

ii) (3,41/5)

iii)LNP is an isosceles triangle (LN = PL = 5, NP =4)

3. i) Midpoint of AC = Midpoint of BD

Central midfielder is at (3,5)

ii) GK + HK = GH,so collinear

ii) equidistant, Cis not the mid point

B is the midpoint of AE, E = (3,-10)

4. i)coordinates of I are $(\frac{28}{5}, \frac{16}{5})$

ii) ratio 3:1

5. i) Distance AB = 4√2

ii) Midpoint = (7,7)

CHAPTER 08 -INTRODUCTION TO TRIGONOMETRY

MULTIPLE CHOICE QUESTIONS

1. $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$ is equal to: (AO1)

(a) –1 (b) 1 (c) 0 (d) 2

2. In Δ ABC right angled at B, sin A = $\frac{7}{25}$, then the value of Cos C is ..... (AO2)

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) $\frac{7}{24}$ (d) $\frac{24}{7}$

3. If 5 tan θ = 4, then the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$ is (AO2)

(a) 1/6 (b) 1/7 (c) 1/4 (d) 1/5

4. If cosec A = 13/12, then the value of $\frac{2\sin A - 3\cos A}{4\sin A - 9\cos A}$ (AO2)

(a) 4 (b) 5 (c) 6 (d) 3

5. Given that sin α = 1/2 and cos β = 1/2, then the value of (β – α) is (AO1)

(a) 0° (b) 30° (c) 60° (d) 90°

6. If tan θ = 1, then the value of sec θ + cosec θ is: (AO1)

(a) $3\sqrt{2}$ (b) $4\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\sqrt{2}$

7. If sin 2A = $\frac{1}{2}\tan^2 45°$ where A is an acute angle, then the value of A is (AO2)

(a) 60° (b) 45° (c) 30° (d) 15°

8. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is (AO2)

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $\sqrt{2}$

ASSERTION AND REASONING QUESTIONS

In the following questions 9 and 12, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. **Assertion (A):** If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$ then the value of $x + y = 3$. (AO1)
Reason (R): For any value of θ, $\sin^2\theta + \cos^2\theta = 1$

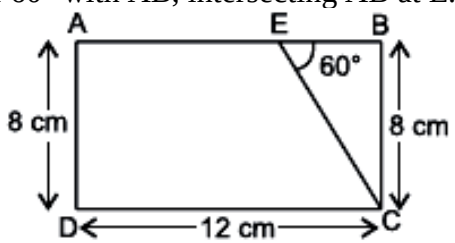
10. **Assertion (A):** Sin A is the product of Sin and A. (AO2)
Reason (R): The value of $\sin\theta$ increases as θ increases.

11. **Assertion (A):** In a right ΔABC, right angled at B, if $\tan A = 1$, then $2 \sin A. \cos A = 1$.
Reason (R): $\tan 45° = 1$ and $\sin 45° = \cos 45° = 1/\sqrt{2}$ (AO1)

12. **Assertion (A):** $\sin (A + B) = \sin A + \sin B$ (AO1)
Reason (R): For any value of θ, $1 + \tan^2\theta = \sec^2\theta$

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

13. In the given figure, ABCD is a rectangle with AD = 8 cm and CD = 12 cm. Line segment CE is drawn, making an angle of 60° with AB, intersecting AB at E. Find the length of CE and BE.



(AO2)

14. If $\sin (A + B) = \sqrt{3}/2$ and $\sin (A – B) = \frac{1}{2}$, $0 \le A + B \le 90°$ and $A > B$, then find A and B. (AO2)

15. Evaluate: $3 \cos^2 60° \sec^2 30° – 2 \sin^2 30° \tan^2 60°$. (AO1)

16. Simplify: $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$ (AO1)

17. If $7 \sin^2 A + 3 \cos^2 A = 4$, then find tan A (AO2)

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

18. If $\operatorname{cosec}\theta + \cot\theta = p$, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$ (AO2)

19. Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \sec\theta + \tan\theta$ (AO2)

20. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$. (AO2)

21. Prove that: $\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^2\theta}{1-\cot\theta}=1+\sin\theta\cos\theta$ (AO2)

22. If cos θ + sin θ = √2 cos θ, show that cos θ – sin θ = √2 sin θ. (AO2)

05 MARKS QUESTIONS

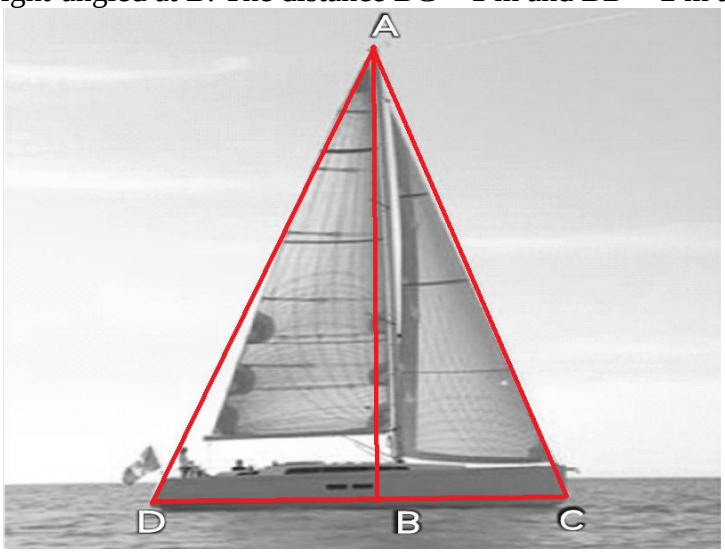
23. If cosec θ – sin θ = m and sec θ – cos θ = n, prove that $(m^2n)^{2/3} + (mn^2)^{2/3} = 1$. (AO2)

24. (a) Find the value of x if tan $3x$ = sin 45°. cos 45° + sin 30°. (2) (AO1)
(b) If sin θ + cos θ = p and sec θ + cosec θ = q, show that $q(p^2 - 1) = 2p$. (3) (AO2)

25. (a) Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ [3] (AO1)
(b) If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\sin\theta$ then find $x^2 + y^2$. [2] (AO2)

CASE STUDY BASED QUESTIONS (4 MARKS EACH)

26. A sailing boat with triangular masts is shown below. Two right triangles can be observed. Triangles ABC and ABD, both right-angled at B. The distance BC = 1 m and BD = 2 m and height AB = 4 m.



Based on the given in formation, answer the following questions:

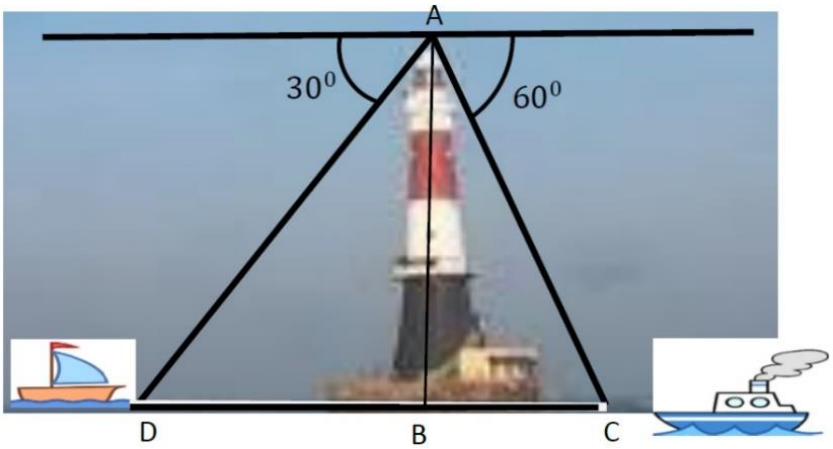
(a) Find the value of sec D. [1] (AO1)

(b) Find the value of cosec C. [1] (AO1)

(c) Find the value of tan D + cot C. [1] (AO2)

(d) Find the value of $\sin^2 C + \cos^2 D$ [1] (AO2)

27. A lighthouse is a tall tower with light near the top. These are often built on islands, coasts or on cliffs. Lighthouses on water surface act as a navigational aid to the mariners and send warning to boats and ships for dangers. Initially wood, coal would be used as illuminators. Gradually it was replaced by candles, lanterns, electric lights. Nowadays they are run by machines and remote monitoring. Prongs Reef lighthouse of Mumbai was constructed in 1874-75. It is approximately 40 meters high and its beam can be seen at a distance of 30 kilometres. A ship and a boat are coming towards the lighthouse from opposite directions. Angles of depression of flash light from the lighthouse to the boat and the ship are 30^0 and 60^0 respectively. (AO2)



(i) Which of the two, boat or the ship is nearer to the light house.

Find its distance from the lighthouse? (2)

(ii) Find the time taken by the boat to reach the light house if it is moving at

the rate of 2 km per hour. (2)

OR

(ii) The ratio of the height of a light house and the length of its shadow on the ground is $\sqrt{3} : 1$. What is the angle of elevation of the sun? (2)

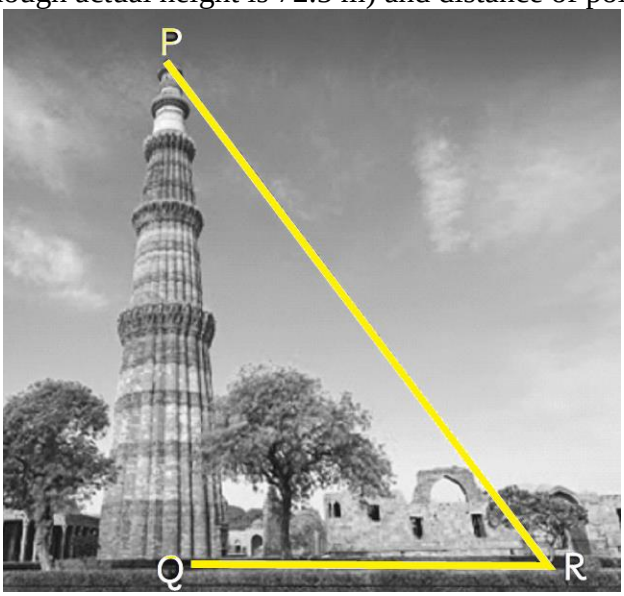
28. An electrician wanted to repair a street lamp at a height of 15 feet. He places his ladder such that its foot is 8 feet from the foot of the lamp post as shown in the figure below: (AO1)

(a) Find the value of cos R.

(b) Find the value of cosec P.

(c) Find the value of cot P – cosec R. (AO2)

29. Built in the 13th century, the magnificent Qutub-Minar in Delhi, in red and buff sandstone is the highest tower in India. It is an architectural marvel of Ancient India. Qutub-ud-Din Aibak of Slave Dynasty laid the foundation of Minar in A.D. 1199 for the use of mu'azzin (crier) to give calls for prayer and raised the first storey, to which were added three more storeys by his successor and son-in-law, Shams-ud-Din Iltutmish (A.D. 1211-36). Let us take the height PQ of Qutub Minar as 72 m for ease of calculations (though actual height is 72.5 m) and distance of point R from Q as 65 m.



(a) Find the value of cos R. (AO1)

(b) Find the value of cosec P – cot P. (AO1)

(c) Find the value of $\frac{6}{\sin R}+\frac{13}{\sin P}-12\tan P$ (AO2)

30. Varanasi is a city of temples, including the gold-plated Vishwanath temple of Lord Shiva; the Bharat Mata, or Mother India, temple that boasts a huge three dimensional relief map of the Indian subcontinent carved out of marble; and the hundreds of small temples that dot the waterways and alleys. It is a city of scholars, home to one of Asia's largest universities. It is also a city of legends. The figure below shows one such temple along the banks of the sacred river "Ganges" or "Ganga". A person sitting at point marked A looks at the top of a nearby temple and imagines that a right angled triangle ABC can be drawn as shown in the figure below.

Based on the above information, answer the following questions. (Take √3 =1.732)

(a) Find the value of sin A. [1] (AO1)

(b) Find the value of sin C. [1] (AO1)

(c) Find the value of tan A – cot C. [1] (AO2)

(d) Find the value of cosec²C. [1] (AO1)

Ans. (a) In ΔABC, sinA = BC/AC

SOLUTIONS: CHAPTER 08 -INTRODUCTION TO TRIGONOMETRY

MULTIPLE CHOICE QUESTIONS

1. (b) 1

2. (a) $\frac{7}{25}$

3. (a) 1/6
4. (d) 3
5. (b) 30°
6. (c) $2\sqrt{2}$
7. (d) 15°
8. (c) $\frac{\sqrt{2}}{2}$

ASSERTION AND REASONING QUESTIONS

9. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
10. (d) Assertion (A) is false but reason (R) is true.
11. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
12. (d) Assertion (A) is false but reason (R) is true.

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

13. In ΔCBE, we have $\tan 60^0 = \frac{CB}{BE}$

$$\Rightarrow \sqrt{3} = \frac{8}{BE} \Rightarrow BE = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3} cm$$

and $\sin 60^0 = \frac{CB}{CE} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{CE} \Rightarrow CE = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3} cm$

14. sin(A + B) = √3/2 = sin 60º
⇒ A + B = 60º .....(i)

sin (A - B) = 1/2 = sin 30º

⇒ A - B = 30º .....(ii)

Solving eq. (i) and (ii), A = 45º and B = 15º

15. $3 \cos^2 60° \sec^2 30° – 2 \sin^2 30° \tan^2 60°$

$$= 3\left(\frac{1}{2}\right)^2\left(\frac{2}{\sqrt{3}}\right)^2 - 2\left(\frac{1}{2}\right)^2\left(\sqrt{3}\right)^2 = \frac{3}{4}\times\frac{4}{3} - 2\times\frac{1}{4}\times 3 = 1 - \frac{3}{2} = -\frac{1}{2}$$

16. $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\cos ec^2\theta}$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1} + \frac{\cos^2\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{1} = \sin^2\theta + \cos^2\theta = 1$$

17. Given, $7\sin^2 A + 3\cos^2 A = 4$
Dividing both sides by $\cos^2 A$, we get

$7 \tan 2A + 3 = 4 \sec^2 A$ [$\because \sec^2\theta = 1 + \tan^2\theta$]

$\Rightarrow 7 \tan^2 A + 3 = 4(1 + \tan^2 A)$

$\Rightarrow 7 \tan^2 A + 3 = 4 + 4 \tan^2 A$

$\Rightarrow 3\tan^2 A = 1 \Rightarrow \tan^2 A = 1/3 \Rightarrow \tan A = 1/\sqrt{3}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

18. Given $\operatorname{cosec}\theta + \cot\theta = p$ ….. (1)

$$\Rightarrow (cosec\theta - \cot\theta)(cosec\theta + \cot\theta) = 1 \Rightarrow (cosec\theta - \cot\theta)p = 1$$

$$\Rightarrow \cos ec\theta - \cot\theta = \frac{1}{p} \quad \text{……. (2)}$$

Adding (1) and (2), we get

$$cosec\theta = \frac{p + \frac{1}{p}}{2} = \frac{p^2+1}{2p}; \cot\theta = \frac{p - \frac{1}{p}}{2} = \frac{p^2-1}{2p}$$

$$\text{Now, } \cos\theta = \frac{\cot\theta}{\cos ec\theta} = \frac{\frac{p^2-1}{2p}}{\frac{p^2+1}{2p}} = \frac{p^2-1}{p^2+1}$$

19. LHS = $\dfrac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ (Dividing numerator and denominator by cos□)

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta + 1 - \sec\theta}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta + 1 - \sec\theta} = \sec\theta + \tan\theta = \text{RHS}$$

20. $\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

$$\Rightarrow 1+2\sin\theta\cos\theta = 3 \Rightarrow 2\sin\theta\cos\theta = 2$$

$$\Rightarrow \sin\theta\cos\theta = 1 = \sin^2\theta + \cos^2\theta$$

$$\Rightarrow 1 = \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \tan\theta + \cot\theta \;\; \Rightarrow \tan\theta + \cot\theta = 1$$

21. $LHS = \dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^2\theta}{1-\cot\theta}$

$$= \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta} = \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)}{\cos\theta-\sin\theta}$$

$$= \cos^2\theta+\sin^2\theta+\cos\theta\sin\theta \;\; = 1+\sin\theta\cos\theta = RHS$$

22. Given, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Squaring both sides, we get

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta$

$\Rightarrow 2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

$\Rightarrow 2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\sqrt{2}\cos\theta)$

$\Rightarrow \sqrt{2}\sin\theta = \cos\theta - \sin\theta \Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

5 MARKS QUESTIONS

23. $\operatorname{cosec}\theta - \sin\theta = m$

$$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = m \Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = m \Rightarrow \frac{\cos^2\theta}{\sin\theta} = m \;\; ...(i)$$

Also, $\sec\theta - \cos\theta = n$

$$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = n \Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = n \Rightarrow \frac{\sin^2\theta}{\cos\theta} = n \;\; ...(ii)$$

Now, LHS = $(m^2n)^{2/3} + (mn^2)^{2/3}$

$$= \left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)\right\}^{2/3} + \left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\right\}^{2/3}$$

$$= \left(\frac{\cos^4\theta.\sin^2\theta}{\sin^2\theta.\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta.\sin^4\theta}{\sin\theta.\cos^2\theta}\right)^{2/3}$$

$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3} = \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$

24. (a) $\tan 3x = \sin 45° . \cos 45° + \sin 30°$

$\Rightarrow \tan 3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \tan 3x = 1 = \tan 45°$

$\Rightarrow 3x = 45° \Rightarrow x = 15°$

(b) $\sin\theta + \cos\theta = p$, $\sec\theta + \text{cosec}\,\theta = q$

LHS $= q(p^2 - 1) = (\sec\theta + \text{cosec}\,\theta)\,[(\sin\theta + \cos\theta)^2 - 1]$

$= \left[\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right] [\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta - 1]$

$= \left[\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}\right] [1 + 2\cos\theta\sin\theta - 1]$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\cos\theta\sin\theta = 2(\sin\theta + \cos\theta) = 2p$ $(\because \sin\theta + \cos\theta = p)$

LHS = RHS. Hence proved.

25. (a) L.H.S $= (\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2\sin A\,\text{cosec}A + \cos^2 A + \sec^2 A + 2\cos A\sec A$

$= \sin^2 A + \cos^2 A + \text{cosec}^2 A + \sec^2 A + 2\sin A \times 1/\sin A + 2\cos A \times 1/\cos A$

(Since, $\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\text{cosec}^2 A = 1 + \cot^2 A$)

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2 = 7 + \tan^2 A + \cot^2 A = \text{RHS}$

(b)

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$

Now, $x\sin\theta = y\cos\theta \Rightarrow \cos\theta\sin\theta = y\cos\theta \Rightarrow y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

CASE STUDY BASED QUESTIONS (4 MARKS EACH)

26. (a) In ΔABD, sec D = AD/BD

by using Pythagoras theorem in right triangle ABD.

$AD^2 = BD^2 + AB^2 = 2^2 + 4^2 = 20 \Rightarrow AD = \sqrt{20} = 2\sqrt{5}$m

∴ sec D = AD/BD = $2\sqrt{5}/2 = \sqrt{5}$

(b) In ΔABC, cosec C = AC/AB

by using Pythagoras theorem in right triangle ABC.

$AC^2 = AB^2 + BC^2 = 4^2 + 1^2 = 17 \Rightarrow AC = \sqrt{17}$ m

∴ cosec C = AC/AB = $\sqrt{17}/4$

(c) In ΔABD, tanD = AB/BD = 4/2 = 2

In ΔABC, cotC = BC/AB = 1/4

∴ tanD + cotC = 2 + 1/4 = 9/4

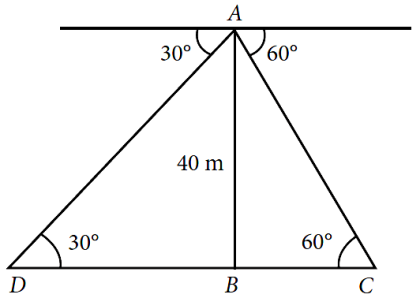
(d) In ΔABC, sinC = AB/AC = $4/\sqrt{17}$

In ΔABD, cosD = BD/AD = $1/\sqrt{5}$

∴ $\sin^2 C + \cos^2 D = 16/17 + 1/5 = 97/85$

27.

(i) Here, height of lighthouse (AB) = 40 m (Given)



In ΔACB, $\tan 60° = \dfrac{AB}{BC}$

$$\Rightarrow \sqrt{3} = \frac{40}{BC} \Rightarrow BC = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3}\text{m}$$

Also, in ΔADB, $\tan 30° = \dfrac{AB}{BD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{DB} \Rightarrow DB = 40\sqrt{3}\text{ m}$$

Thus, ship is nearer to the light house.

(ii) Boat moving at the speed of 2 km/hr *i.e.*, $\dfrac{2000}{60}$ m/min.

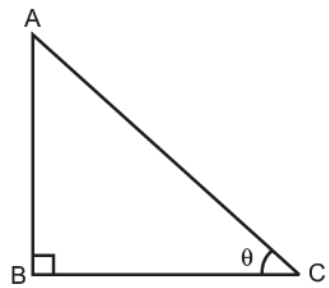
$$\therefore \text{ Time taken to cover the distance} = \frac{\text{Distance } DB}{\text{Speed}} = \frac{60}{2000} \times 40\sqrt{3} = 2.078 \text{ minutes}$$

OR

(ii) Let height of light house be AB and its shadow be BC.

In ΔABC, $\tan\theta = \frac{AB}{AC}$

But $\frac{AB}{AC} = \frac{\sqrt{3}}{1} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3} = \tan 60^0 \Rightarrow \theta = 60^0$



28. (a) We will first calculate PR by using Pythagoras theorem in ΔPQR.
So, $PR^2 = PQ^2 + QR^2 = (15)^2 + 8^2 = 225 + 64 = 289 = 17^2$

⇒ PR = 17 feet

In ΔPQR, $\cos R = \frac{Adj}{Hyp} = \frac{QR}{PR} = \frac{8}{17}$

(b) In ΔPQR, $\cos ecP = \frac{Hyp}{Opp} = \frac{PR}{QR} = \frac{17}{8}$

(c) In ΔPQR, $\cot P = \frac{Adj}{Opp} = \frac{PQ}{QR} = \frac{15}{8}$

$\cos ecR = \frac{Hyp}{Opp} = \frac{PR}{PQ} = \frac{17}{15}$

Now, $\cot P - \cos ecR = \frac{15}{8} - \frac{17}{15} = \frac{225-136}{120} = \frac{89}{120}$

29. (a) We will apply Pythagoras theorem in right triangle PQR to find PR.
$PR^2 = PQ^2 + QR^2 = 72^2 + 65^2 = 9409 = 97^2$

⇒ PR = 97 m

In ΔPQR, $\cos R = \frac{Adj}{Hyp} = \frac{QR}{PR} = \frac{65}{97}$

(b) In ΔPQR, $\cos ecP = \frac{Hyp}{Opp} = \frac{PR}{QR} = \frac{97}{65}$

$\cot P = \frac{Adj}{Opp} = \frac{PQ}{QR} = \frac{72}{65}$

Now, $\cos ecP - \cot P = \frac{97}{65} - \frac{72}{65} = \frac{25}{65} = \frac{5}{13}$

(c) $\sin R = \frac{PQ}{PR} = \frac{72}{97}, \sin P = \frac{QR}{PR} = \frac{65}{97}, \tan P = \frac{QR}{PQ} = \frac{65}{72}$

$$\frac{6}{\sin R} + \frac{13}{\sin P} - 12\tan P = \frac{6}{\frac{72}{97}} + \frac{13}{\frac{65}{97}} - 12 \times \frac{65}{72}$$

$$= 6 \times \frac{72}{97} + 13 \times \frac{65}{97} - 12 \times \frac{65}{72} = \frac{97}{12} + \frac{97}{5} - \frac{65}{6}$$

$$= \frac{97 \times 5 + 97 \times 12 - 65 \times 10}{60} = \frac{999}{60} = \frac{333}{20}$$

30. (a) In ΔABC, sinA = BC/AC

by using Pythagoras theorem in right triangle ABC.

$AC^2 = AB^2 + BC^2 = 12^2 + 5^2 = 144 + 25 = 169 \Rightarrow AD = 13$ m

∴ sinA = BC/AC = 5/13

(b) In ΔABC, sinC = AB/AC ⇒ sinC = AB/AC = 12/13

(c) In ΔABC, tanA = BC/AB = 5/12 ⇒ cotC = BC/AB = 5/12

Therefore, tan A – cot C = 0

(d) In ΔABC, sinC = AB/AC = 12/13

cosecC = 1/sinC = 13/12

Therefore, $cosec^2C = 169/144$

CHAPTER 09 - SOME APPLICATION OF TRIGONOMETRY

MULTIPLE CHOICE QUESTIONS

1. If 300 m high pole makes an angle of elevation at a point on ground which is 300 m away from its foot, then the angle of elevation is: **(AO1)**

(a) 60° (b) 90° (c) 30° (d) 45°

2.The angle of depression of a bike parked on the road from the top of a 90 m high pole is 60 degrees. The distance of the bike from the pole is: **(AO1)**

(a) $20\sqrt{3}$ m (b) 90 m (c) $15\sqrt{3}$ m (d) $30\sqrt{3}$ m

3.A stone is $15\sqrt{3}$ m away from a tower 15 m high, then the angle of elevation of the top of the tower from the stone is: **AO1)**

(a) 45° (b) 60° (c) 30° (d) 90°

4.The ratio of the length of a tower and its shadow is $\sqrt{3} : 1$. The altitude of the sun is: **(AO1)**

(a) 0° (b) 60° (c) 30° (d) 45°

5.The tops of the poles of height 16 m and 10 m are connected by a wire of length x meters. If the wire makes an angle of 30° with the horizontal, then x = **(AO2)**

(a) 26 m (b) 16 m (c) 12 m (d) 10 m

6.The tops of two poles of heights 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is **(AO2)**

(a) 8 m (b) 10 m (c) 12 m (d) 14 m

7.If the angle of depression of an object from a temple is 30°, and the distance of the object from the temple is 45 m, then the height of the temple is: **(AO1)**

(a) $45\sqrt{3}$ m (b) $15\sqrt{3}$ m (c) 20 m (d) $20\sqrt{3}$ m

8.If two towers of heights h_1 and h_2 subtend angles of 60° and 30° respectively at the mid-point of the line joining their feet, then $h_1 : h_2$ = **(AO2)**

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

ASSERTION AND REASON QUESTIONS

In the following questions 9 and 12, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9.**Assertion (A):** If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45º. **(AO2)**

Reason (R): According to Pythagoras theorem, $h^2 = l^2 + b^2$, where h = hypotenuse, l = length and b = base.

10.Assertion (A): The ladder 20 m long makes an angle 60° with the wall, then the height of the point where the ladder touches the wall is 15 m. **(AO1)**

Reason (R): For an angle θ, $\cos\theta = \frac{Adjacent\ \ Side}{Hypotenuse}$

11.Assertion (A): The joker in a circus climb a rope of 10 m long which is tied from the top of a vertical pole to the ground. The angle made by the rope with the ground level is 30°, then the height of the pole is 5 m.

Reason (R): For an angle θ, $\sin\theta = \frac{Opposite\ \ Side}{Hypotenuse}$ **(AO1)**

12.Assertion (A): The length of the shadow of a tree 20 m long is 20√3 m, when the sun's angle of elevation is
30°.

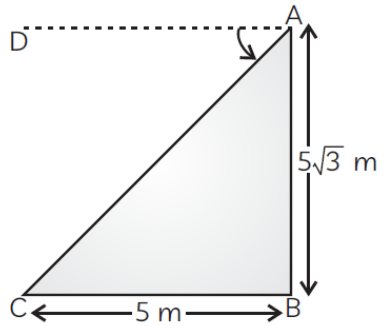
Reason (R): For an angle θ, $\cos\theta = \frac{Adjacent\ \ Side}{Hypotenuse}$ **(AO1)**

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

13.The angle of depression of a car standing on the ground, from the top of a 85 m high tower is 45º. Find the distance of the car from the base of the tower. **(AO1)**

14.A pole casts a shadow of length 2√3 m on ground, when the sun's elevation is 60°. Find the height of the pole. **(AO2)**

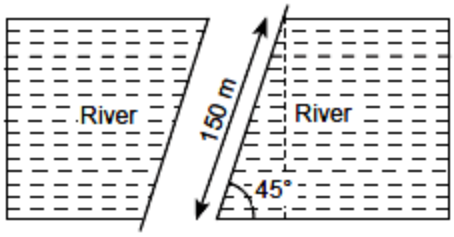
15.The figure shows the observation of point C from point A. Find the angle of depression from A.



(AO2)

16.The shadow of a flagstaff is three times as long as the shadow of the flagstaff when the sunrays meet the ground at an angle of 60°. Find the angle between the sunrays and the ground at the time of longer shadow. **(AO2)**

17.A bridge across a river makes an angle of 45° with the river bank (Fig. given). If the length of the bridge across the river is 150 m, what is the width of the river? **(AO2)**



SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

18.A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30°. Find the speed of the boat. (Use $\sqrt{3}$ = 1.732) **(AO2)**

19.A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732] **(AO1)**

20.As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30º and 45º. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use √3 = 1.732] **(AO1)**

21.A boy 1.7 m tall is standing on a horizontal ground, 50 m away from a building. The angle of elevation of the top of the building from his eye is 60°. Calculate the height of the building. (Take √3 = 1.73) **(AO1)**

22.Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men. (Use √3 = 1.73) **(AO2)**

5 MARKS QUESTIONS

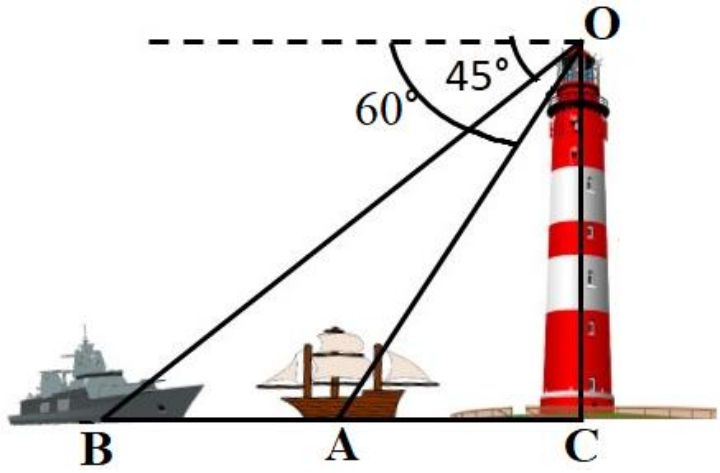
23.At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A. **(AO2)**

24.The angles of depression of the top and bottom of a 12 m tall building, from the top of a multi-storeyed building are 30° and 60° respectively. Find the height of the multi-storeyed building. **(AO2)**

25.An aero plane flying at a height of 9000 m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two planes from the same point on the ground are 60° and 30° respectively. Find the vertical distance between the aero planes at that instant. **(AO2)**

CASE STUDY BASED QUESTIONS (4 MARKS)

26.A person/observer on the sea coast observes two ships in the sea, both the ships are in same straight path one behind the other. If the observer is on his building of height 20 meters (including observer) and he observes the angle of depression of two ships as 45° and 60° respectively. **(AO1)**



On the basis of above information answer the following questions.

(i) If a person observes a ship whose angle of depression is 60° then how much distance is the ship away from the building?

(ii) If a person observes another ship whose angle of depression is 45° then how much distance that ship is away from the building?

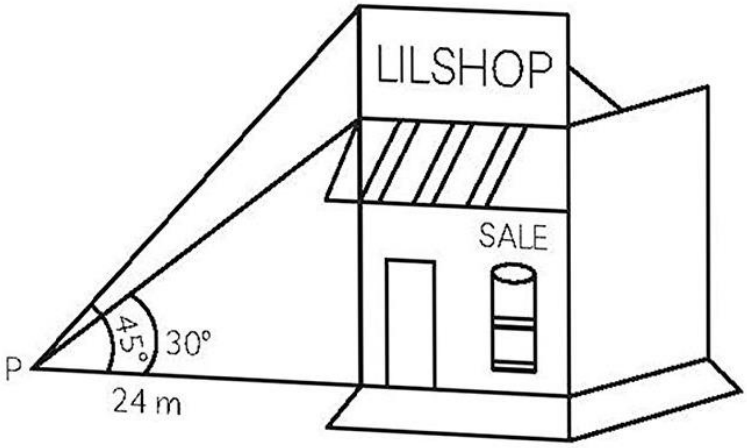
(iii) If a person observes the ship whose angle of depression changes from 60° to 30° then how far be ship from the building if the observer is at 20 m of height (including him)?

OR

At a time when a person observes two ships whose angle of depressions are 60° and 45° the distance between the ships is (in meter).

27.Anita purchased a new building for her business. Being in the prime location, she decided to make some more money by putting up an advertisement sign for a rental ad income on the roof of the building.

From a point P on the ground level, the angle of elevation of the roof of the building is 30° and the angle of elevation of the top of the sign board is 45°. The point P is at a distance of 24 m from the base of the building. **(AO2)**



On the basis of the above information, answer the following questions:

(i) Find the height of the building (without the sign board). (2)

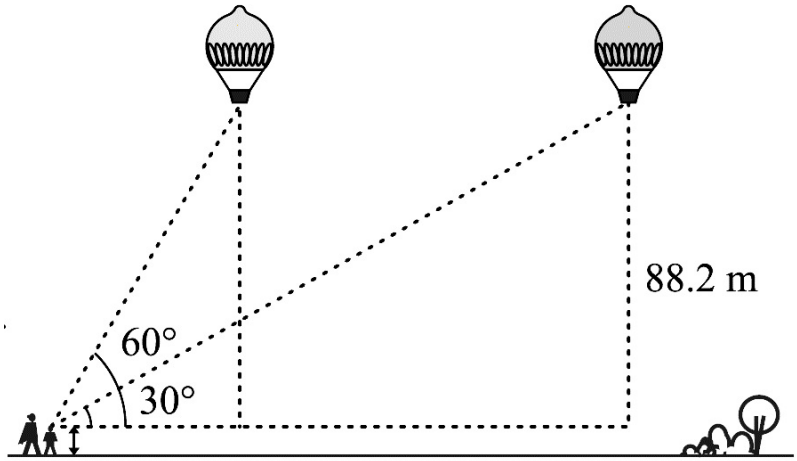
OR

Find the height of the building (with the sign board) (2)

(ii) Find the height of the sign board. (1)

(iii) Find the distance of the point P from the top of the sign board. (1)

28.A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After 30 seconds, the angle of elevation reduces to 30° (see the below figure). **(AO2)**



Based on the above information, answer the following questions. (Take √3 =1.732)

(i) Find the distance travelled by the balloon during the interval. (2)

(ii) Find the speed of the balloon. (2)

OR

(ii) If the elevation of the sun at a given time is 30°, then find the length of the shadow cast by a tower of 150 feet height at that time. (2)

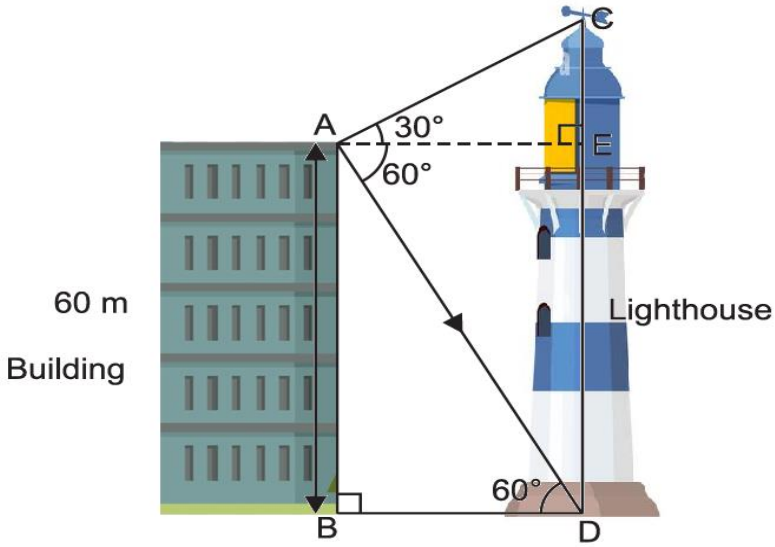
29.Ram is watching the top and bottom of a lighthouse from the top of the building. The angles of elevation and depression of the top and bottom of a lighthouse from the top of a 60 m high building are 30° and 60° respectively. **(AO2)**

Find (i) the difference between the heights of the lighthouse and the building.

(ii) the distance between the lighthouse and the building.

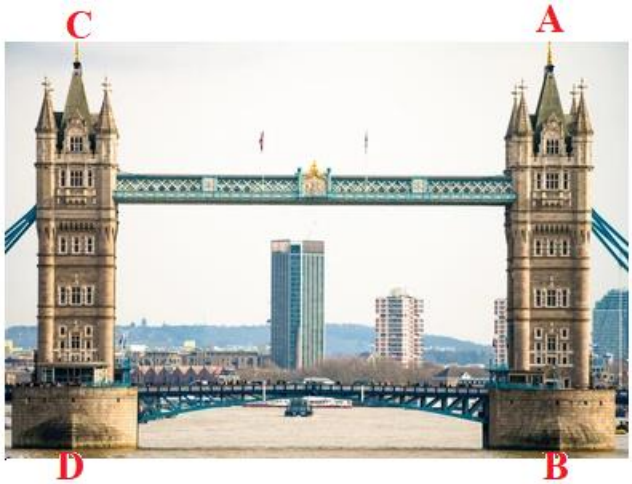
OR

The ratio of the height of a light house and the length of its shadow on the ground is √3 : 1 What is the angle of elevation?



30.Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was 60° and 30° respectively. **(AO1)**



(i) Find the distances of the point from the base of the towers where Neeta was standing while measuring the height. [2]

(ii) Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated? [2]

OR

(ii) Find the distance between Neeta and top of tower AB? Also, Find the distance between Neeta and top tower CD? [2]

****

SOLUTIONS: CHAPTER 09 - SOME APPLICATION OF TRIGONOMETRY

MULTIPLE CHOICE QUESTIONS

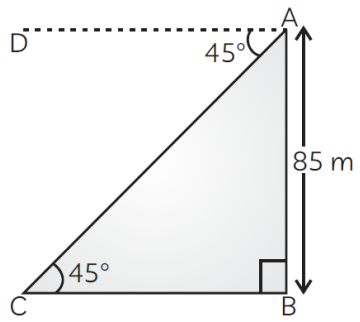
1. (d) 45°
2. (d) 30√3 m
3. (c) 30°
4. (b) 60°
5. (c) 12 m
6. (c) 12 m
7. (b) 15√3 m
8. (d) 3 : 1

ASSERTION AND REASONING QUESTIONS

9. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
10. (d) Assertion (A) is false but reason (R) is true.
11. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
12. (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

SHORT ANSWER TYPE QUESTIONS (2 MARKS QUESTIONS)

13. Let AB = 85 m be the height of the tower and angle of depression is ∠DAC = 45°.
Then, ∠ACB = ∠DAC = 45° [alternate angles]



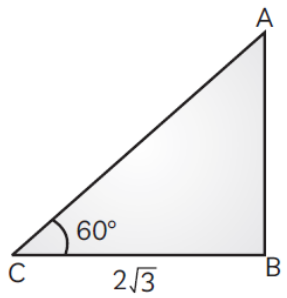
Now, in right–angled ΔABC, tan45° = AB/BC

⇒ 1 = 85/BC

⇒ BC = 85 m

Hence, the distance of the car from the base of the tower is 85 m.

14. A pole casts a shadow of length $2\sqrt{3}$ m on ground, when the sun's elevation is 60°. Find the height of the pole.
Ans. Let AB be the pole and BC be its shadow.



∴ In ABC, tan 60° = AB/BC

⇒ $\sqrt{3}$ = AB/ $2\sqrt{3}$

⇒ AB = 6 m

Hence, the height of the pole is 6 m.

15. In right–angled ΔABC, ∠B = 90°
Let ∠DAC = θ

Then ∠DAC = ∠ACB = θ [alternate angles]

Now, tan θ = AB/BC = $5\sqrt{3}$ / 5

⇒ tan θ = $\sqrt{3}$

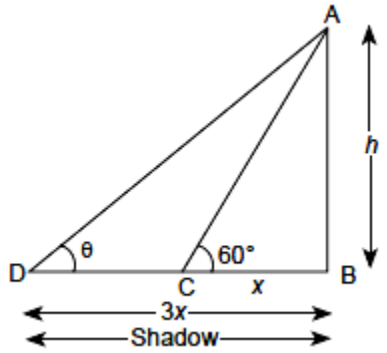
⇒ tan θ = tan 60°

⇒ θ = 60°

Hence, the angle of depression from A is 60º.

16. In ΔABC, tan 60° = $\frac{AB}{BC} = \frac{h}{x}$

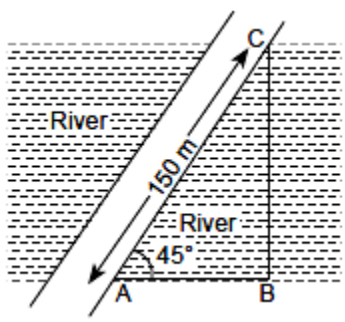
⇒ $\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$



In ΔABD, tan θ = $\frac{AB}{BD} \Rightarrow \tan\theta = \frac{h}{3x}$

$\Rightarrow$ tan θ = $\frac{\sqrt{3}x}{3x} = \frac{1}{\sqrt{3}} = \tan 60^0 \Rightarrow \theta = 60^0$

17. In given figure, $\sin 45^0 = \frac{BC}{AC}$



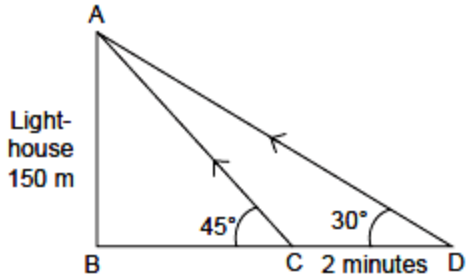
$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{150} \Rightarrow BC = \frac{150}{\sqrt{2}} = 75\sqrt{2}m$

SHORT ANSWER TYPE QUESTIONS (3 MARKS QUESTIONS)

18. Let AB is lighthouse.

∴ AB = 150 m

Initially boat is at C and after 2 minutes it reaches at D.



In right Δ ABC, $\frac{AB}{BC}$ = tan 45°

$\Rightarrow \frac{150}{BC} = 1 \Rightarrow$ BC = 150 m

In right Δ ABD, $\frac{AB}{BD}$ = tan 30°

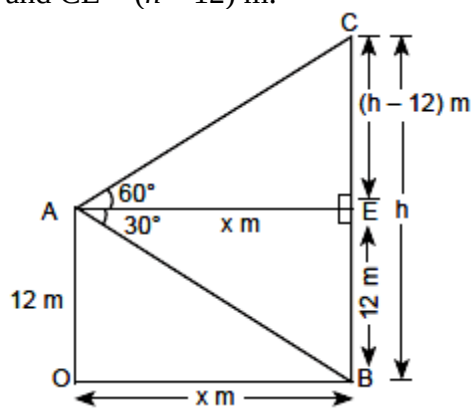
$\Rightarrow \frac{150}{BD} = \frac{1}{\sqrt{3}} \Rightarrow BD = 150\sqrt{3}m$

Distance covered in 2 minutes = BD – BC = 150 $\sqrt{3}$ – 150 = 150 ($\sqrt{3}$ – 1) m

$\therefore$ Speed = $\frac{\text{Distance covered}}{\text{time taken}} = \frac{150(\sqrt{3}-1)}{2}$

= 75 × (1.732 – 1) = 54.9 m/minutes

19. A is the position of the man, OA = 12m, BC is cliff.
Let height of the cliff BC = h m and CE = (h – 12) m.



Let AE = OB = x m

In right angled triangle AEB, $\frac{AE}{BE}$ = cot 30° $\Rightarrow$ AE = 12 × $\sqrt{3}$

= 12 × 1.732 m = 20.78 m.

$\therefore$ Distance of ship from cliff = 20.78 m.

In right angled triangle AEC, $\frac{CE}{AE} = \tan 60^0 \Rightarrow \frac{h-12}{12\sqrt{3}} = \sqrt{3}$

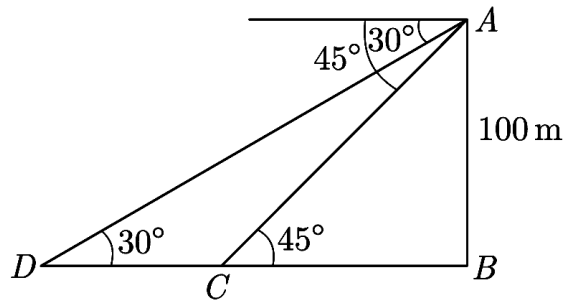
h – 12 = 36 $\Rightarrow$ h = 48 m

$\therefore$ Height of the cliff = 48 m

20. Let AB be the tower and ships are at points C and D. As per question statement we have shown diagram below.

Now in ΔABC we have $\tan 45^\circ = \frac{AB}{AC}$

$$\Rightarrow \frac{AB}{AC} = 1 \Rightarrow AB = BC$$



Now in ΔABD we have $\tan 30^\circ = \frac{AB}{BD}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$

$$\Rightarrow AB + CD = \sqrt{3}\,AB$$

$$\Rightarrow CD = AB(\sqrt{3} - 1) = 100 \times (1.732 - 1) = 73.2 \text{ m}$$

Distance between two ships is 73.2 m.

21. Ans:

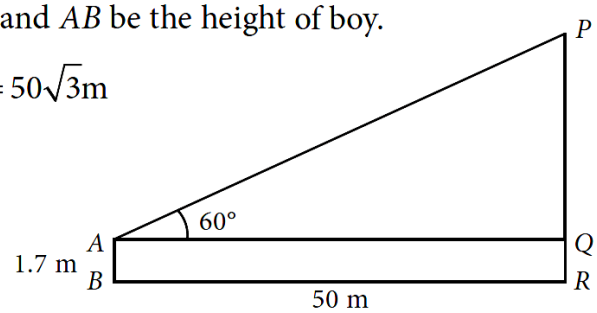
Let PR be the height of building and AB be the height of boy.

In ΔPQA, $\tan 60^\circ = \frac{PQ}{50} \Rightarrow PQ = 50\sqrt{3}\text{m}$

So, height of the building $= PR$

$= (50\sqrt{3} + 1.7)\text{m}$

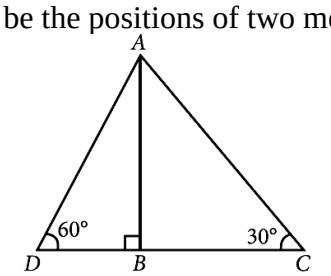
$= 88.2$ m



22. Let AB = 75 m be the building and C, D be the positions of two men.

Now, in ΔABC, $\tan 30^\circ = \frac{AB}{BC}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BC} \Rightarrow BC = 75\sqrt{3} \text{ m}$$



In ΔABD, $\tan 60^\circ = \frac{AB}{BD}$

$$\Rightarrow \sqrt{3} = \frac{75}{BD} \Rightarrow BD = \frac{75}{\sqrt{3}} \text{ m} = 25\sqrt{3} \text{ m}$$

$\therefore$ Distance between the two men $= BC + BD = 75\sqrt{3} + 25\sqrt{3} = 100\sqrt{3} = 173$ m

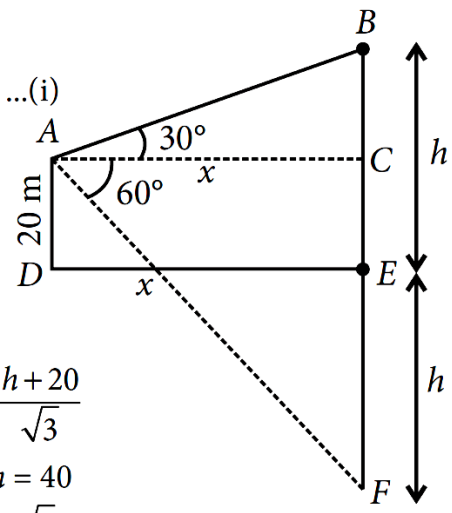
5 MARKS QUESTIONS

23. Let DE be the level of water and cloud be at position B which is h m above the level of water and reflection of cloud be at F and AC = DE = x m.

$\therefore$ BC = (h – 20)m, CF = (h + 20) m

In ΔABC, $\tan 30° = \dfrac{BC}{AC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h-20}{x} \Rightarrow x = \sqrt{3}(h-20)$ ...(i)



In ΔACF,

$\tan 60° = \dfrac{CF}{AC} \Rightarrow \sqrt{3} = \dfrac{h+20}{x}$

$\Rightarrow x = \dfrac{h+20}{\sqrt{3}}$ ...(ii)

From (i) and (ii), we get $\sqrt{3}(h-20) = \dfrac{h+20}{\sqrt{3}}$

$\Rightarrow 3h - 60 = h + 20 \Rightarrow 2h = 80 \Rightarrow h = 40$

From (i), we have $x = \sqrt{3}(40-20) = 20\sqrt{3}$

Applying Pythagoras theorem in ΔABC,

$AB^2 = BC^2 + AC^2 = (20)^2 + (20\sqrt{3})^2$

$= 400 + 1200 = 1600 \Rightarrow AB = \sqrt{1600} = 40$ m

$\therefore$ Distance of the cloud from point A = 40 m

24. Let AB be the building and CD be the multi-storeyed building of height h m.
Here, AB = CE = 12 m and DE = (h – 12) m

In ΔACD, $\tan 60° = \dfrac{CD}{AC} \Rightarrow \sqrt{3} = \dfrac{h}{AC} \Rightarrow AC = \dfrac{h}{\sqrt{3}}$

In ΔBDE, $\tan 30° = \dfrac{DE}{BE} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{(h-12)}{BE}$

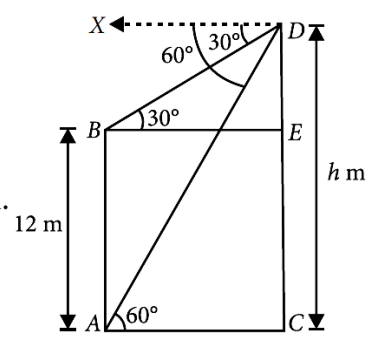
$\Rightarrow BE = (h-12)\sqrt{3}$ m

Now, $AC = BE \Rightarrow \dfrac{h}{\sqrt{3}} = (h-12)\sqrt{3}$

$\Rightarrow h = 3(h-12) \Rightarrow h = 3h - 36$

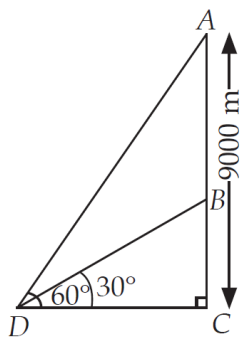
$\Rightarrow -2h = -36 \Rightarrow h = 18$

Thus, the height of the multi-storeyed building is 18 m.



25. Let A and B be the positions of two aero planes when A is vertically above B and AC = 9000 m.
Let D be the point of observation on the ground such that

$\angle ADC = 60°$ and $\angle BDC = 30°$.



In ΔACD, $\tan 60° = \dfrac{AC}{CD}$

$\Rightarrow \quad \sqrt{3} = \dfrac{9000}{CD}$

$\Rightarrow \quad CD = \dfrac{9000}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = 3000\sqrt{3}$ m $\quad \ldots$(i)

In ΔBCD, $\tan 30° = \dfrac{BC}{CD}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{BC}{3000\sqrt{3}}$ [From (i)]

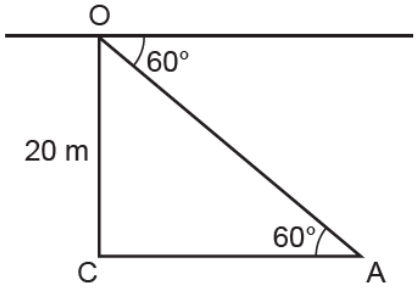
$\Rightarrow \quad BC = 3000$ m

$\therefore$ Vertical distance between A and B, $AB = AC - BC$

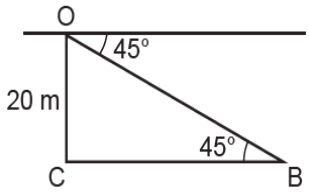
$= 9000 - 3000 = 6000$ m

CASE STUDY BASED QUESTIONS (4 MARKS EACH)

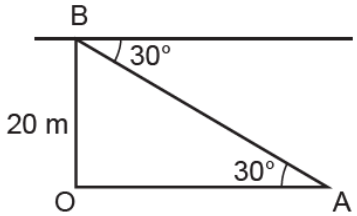
26. (i) $\tan 60^0 = \dfrac{OC}{AC} \Rightarrow \sqrt{3} = \dfrac{20}{AC} \Rightarrow AC = \dfrac{20}{\sqrt{3}} = \dfrac{20\sqrt{3}}{3} m = 11.55m$



(ii) $\tan 45^0 = \dfrac{OC}{BC} \Rightarrow 1 = \dfrac{20}{BC} \Rightarrow BC = 20m$



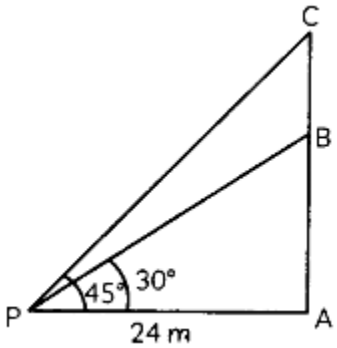
(iii) $\tan 30^0 = \frac{OB}{OA} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{OA} \Rightarrow OA = 20\sqrt{3}m$



OR

Distance between two ships 20 m = BC – AC = 20 – 11.55 = 8.45 m

27. (i) In ΔAPC,



tan 30° = AB/AP

⇒ 1/√3 = AB/24

⇒ AB = 24/√3 m = 13.85 m = 14 m (approx)

OR

Considering, the diagram in the above question, AC as the new height of the shop including the sign-baard.

In ΔAPC,

tan 45° = AC/AP

⇒ 1 = AC/24

⇒ AC = 24 m

(ii) From Q (i) and Q (ii).

Length of sign board, BC = AC – AB

= 24 – 14

= 10 m

(iii) In ΔAPC,

cos 45° = AP/AC

⇒ 1/√2 = 24/AC

⇒ PC = 24√2 m

28. (i) In the figure, let C be the position of the observer (the girl).
A and P are two positions of the balloon.

CD is the horizontal line from the eyes of the (observer) girl.

Here PD = AB = 88.2 m − 1.2 m = 87 m

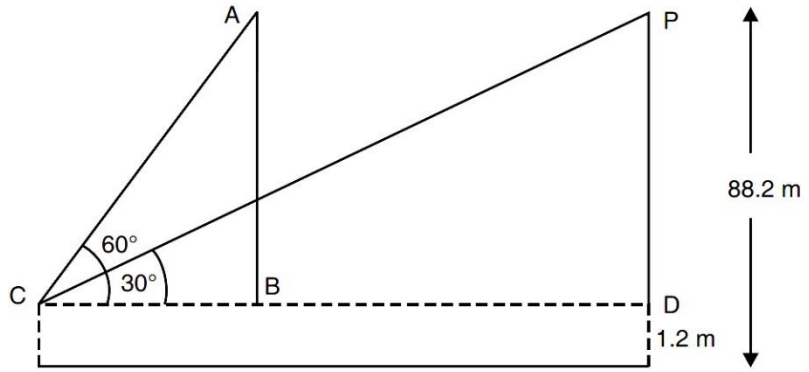
In right $\Delta\, ABC$, we have $\frac{AB}{BC} = \tan 60°$

$\Rightarrow \frac{87}{BC} = \sqrt{3} \quad \Rightarrow \quad BC = \frac{87}{\sqrt{3}}$ m

In right $\Delta\, PDC$, we have $\frac{PD}{CD} = \tan 30°$

$\Rightarrow \frac{87}{CD} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad CD = 87\sqrt{3}$

Now, $BD = CD - BC = 87\sqrt{3} - \frac{87}{\sqrt{3}} = 58\sqrt{3}$ m

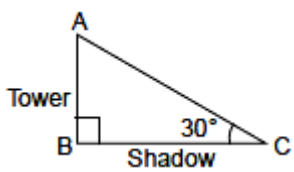


Thus, the required distance between the two positions of the balloon = 58 √3 m

= 58 x 1.732 = 100.46 m (approx.)

(ii) Speed of the balloon = Distance/time = 100.46/30 = 3.35 m/s (approx.)

OR



In right ΔABC

$$\frac{AB}{BC} = \tan 30° \Rightarrow \frac{150}{BC} = \frac{1}{\sqrt{3}} \Rightarrow BC = 150\sqrt{3} \text{ feet}$$

29. In right ΔABD,

$$\tan 60^0 = \frac{AB}{BD} \Rightarrow \sqrt{3} = \frac{60}{BD} \Rightarrow BD = \frac{60}{\sqrt{3}} = 20\sqrt{3}m$$

∴ AE = 20√3 m (∵ BD = AE)

Now in right ΔAEC

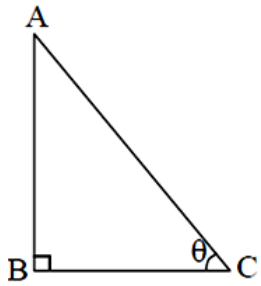
$$\tan 30^0 = \frac{CE}{AE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CE}{20\sqrt{3}} \Rightarrow CE = 20m$$

(i) Difference between the heights of the lighthouse and the building = CE = 20 m

(ii) The distance between the lighthouse and the building = BD = 20√3 m.

OR

Let AB be the light house, BC be its shadow and θ be the angle of elevation of the sun at that instant



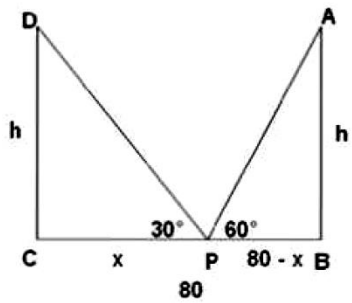
Then, in triangle ABC, we have, tan θ = $\frac{AB}{BC}$

$\tan\theta = \frac{\sqrt{3}}{1} = \tan 60^0 \Rightarrow \theta = 60°$

Hence, angle of elevation of the sun is 60°.

30. (i) Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 - x).
Also, ∠APB = 60° and ∠DPC = 30°

In right angled triangle DCP, $\tan 30^0 = \frac{CD}{CP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow h = \frac{x}{\sqrt{3}}$ …… (1)



In right angled triangle ABP, $\tan 60^0 = \frac{AB}{BP} \Rightarrow \frac{h}{80-x} = \sqrt{3} \Rightarrow h = \sqrt{3}(80-x)$

$\Rightarrow \frac{x}{\sqrt{3}} = \sqrt{3}(80-x)$ ⇒ x = 3(80 – x) ⇒ x = 240 -3x

⇒ x + 3x = 240 ⇒ 4x = 240 ⇒ x=60

Thus, the position of the point P is 60 m from C.

(ii) Height of the tower, h = $\frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

The height of each tower is $20\sqrt{3}$ m.

OR

(ii) The distance between Neeta and top of tower AB.

In ΔABP, $\sin 60^0 = \frac{AB}{AP} \Rightarrow \frac{\sqrt{3}}{2} = \frac{20\sqrt{3}}{AP} \Rightarrow AP = 40m$

Similarly, the distance between Neeta and top of tower CD.

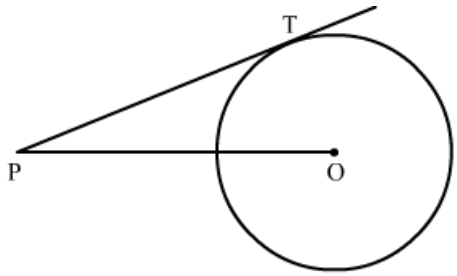
In ΔCDP, $\sin 30^0 = \frac{CD}{PD} \Rightarrow \frac{1}{2} = \frac{20\sqrt{3}}{PD} \Rightarrow PD = 40\sqrt{3}m$

****

CHAPTER 10-CIRCLES

MULTIPLE CHOICE QUESTIONS

1. In the given fig., point P is 26 cm away from the centre O of a circle and the length PT of the tangent drawn from P to the circle is 24 cm. Then the radius of the circle is



(a) 25 cm (b) 26 cm (c) 24 cm (d) 10 cm **(AO1)**

2. A line through point of contact and passing through centre of circle is known as
(a) tangent (b) chord (c) normal (d) segment **(AO1)**

3. C (O, r_1) and C(O, r_2) are two concentric circles with $r_1 > r_2$. AB is a chord of C(O, r_1) touching
a) AB = r_1 (b) AB = r_2 (c) AC = BC (d) AB = $r_1 + r_2$ **(AO1)**

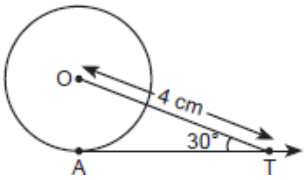
4. Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25π cm^2, then AB is equal to **(AO1)**

(a) 5 cm (b) 8 cm (c) 10 cm (d) 25 cm

5. A circle touches x-axis at A and y-axis at B. If O is origin and OA = 5 units, then diameter of the circle is **(AO1)**

(a) 8 units (b) 10 units (c) 10 $\sqrt{2}$ units (d) 8 $\sqrt{2}$ units

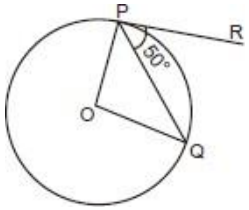
6. In figure AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°. Then AT is equal to **(AO2)**



(a) 4 cm (b) 2 cm (c) 2√3 cm (d) 4√3 cm

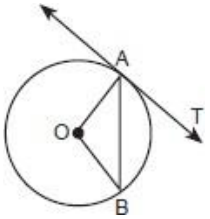
7. In figure if O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle

50° with PQ, then ∠POQ is equal to **(AO2)**



(a) 100° (b) 80° (c) 90° (d) 75°

8. In figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠AOB = 100°, then ∠BAT is equal to **(AO2)**



(a) 100° (b) 40° (c) 50° (d) 90°

ASSERTION AND REASON QUESTIONS

In the following question a statement of Assertion (A) is followed by a statement of reason (R) , choose the correct answer out of the following cases:

A) Both A and R are true and R is the correct explanation of A.

B) Both A and R are true and R is not the correct explanation of A.

C) A is true and R is false.

D) A is false and R is true.

9. **Assertion (A):** The radius of a circle is perpendicular to the tangent at the point of contact.
Reason (R): A tangent to a circle is a line that intersects the circle at exactly two points. (A01)

10**. Assertion (A):** The length of the tangents drawn from an external point to a circle are equal.
Reason (R): Tangents from an external point to a circle make equal angles with the line joining the center to that point. (AO1)

11. **Assertion (A):** The tangents drawn at the ends of a diameter of a circle are parallel. (AO2)

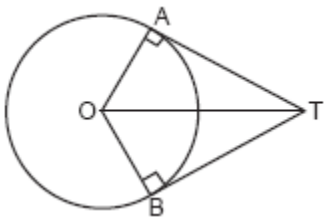
Reason (R): The angle between the tangent and the radius at the point of contact is 90°

12**. Assertion (A):** If two tangents are drawn to a circle from an external point, then the angles between the tangents and the chord joining the points of contact are equal.

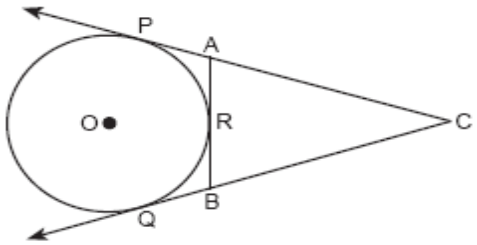
Reason (R): The tangents drawn from an external point to a circle are always equal in length. (AO2)

2 MARKS QUESTIONS

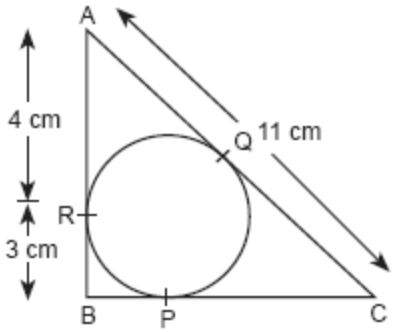
13. In figure if ∠ATO = 40°, find ∠AOB. **(AO1)**



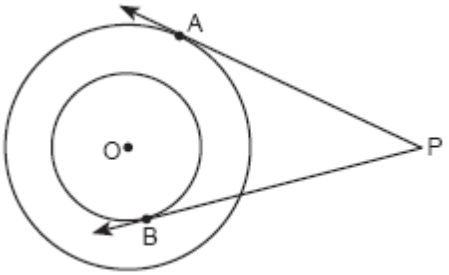
14. In figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm, and BC = 7 cm, then find the length of BR. **(AO1)**



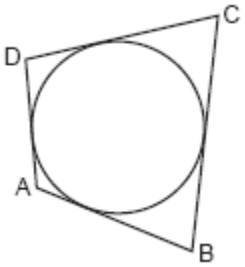
15. In figure, ΔABC is circumscribing a circle. Find the length of BC. **(AO1)**



16. In figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP. **(AO1)**



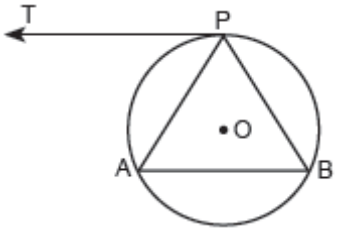
17. In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD. **(AO1)**



<u>3 MARKS QUESTIONS</u>

18. A tangent PT is drawn parallel to a chord AB as shown in figure. Prove that APB is an isosceles triangle.

(AO2)

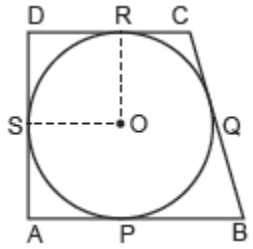


19. AB is diameter and AC is a chord of a circle such that $\angle BAC = 30°$. If tangent at C intersects AB produced in D, prove that BC = BD. **(AO2)**

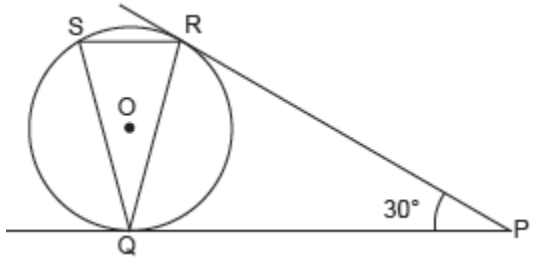
20. In a right triangle ABC, a circle with a side AB as diameter is drawn to intersect the

hypotenuse AC at P. Prove that the tangent to the circle at P bisects the side BC. **(AO2)**

21. A quadrilateral ABCD is drawn so that ∠D = 90°, BC = 38 cm and CD = 25 cm. A circle is inscribed in the quadrilateral and it touches the side AB, BC, CD and DA at P, Q, R and S respectively. If BP = 27 cm, find the radius of the inscribed circle. **(AO2)**

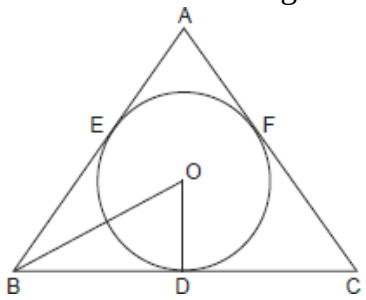


22. In figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS. **(AO2)**



5 MARKS QUESTIONS

23. A circular region is inscribed in a triangular boundary as shown in figure. **(AO2)**



Each boundary of triangular part is act as tangent to the circle, where O is centre of circle and OD ⊥BC.

Answer the questions based on above

(a) What will be the radius of the circle, if BD = 24 cm and OB = 25 cm?

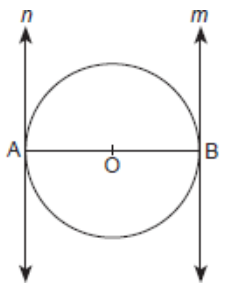
(b) Determine CD, if OC = 26 cm.

(c) As AB and AC act as tangents to the circle at E and F and AE = 8 cm, then what is the perimeterof ΔABC.

(d) Determine area of ΔBOC.

(e) What is the area of ΔABC?

24. A circle can have at most two parallel tangents, one at a point on it and the other at a point diametrically opposite to it. Here AB is diameter of a circle and tangent n and tangent m are parallel to each other. Answer the questions based on above. **(AO2)**



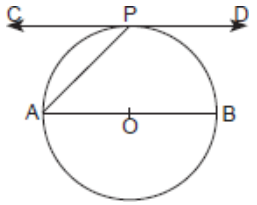
(a) What is the distance between two parallel tangents of a circle of radius 6 cm.

(b) Two parallel tangents touches the circle at points A and B. Find the distance between paralleltangents if area of circle is 25 π cm^2.

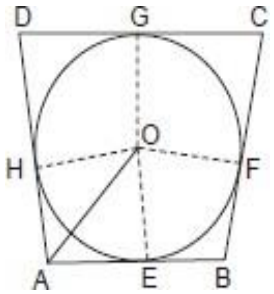
(c) What is the maximum number of parallel tangents a circle can have?

(d) CD is tangent to circle at P. If ∠PAB = 30°. Determine ∠CPA, where O is centre of circle.

(e) How many tangents can be drawn to a circle from a point lying inside the circle?



25. A welfare society of birds constructed a circular tank to serve as a bird bath as shown in figure



Here ABCD is a quadrilateral sides AB, BC, CD, DA act as tangents to circle at E, F, G and H. Here AB = 5 m, CD = 6 m and BC = 7 m **(AO2)**

Answer the questions based on above.

(a) Distance BC =

(i)11 m (ii) 4 m (iii) 7 m (iv) 6 m

(b) If O is centre of tank and AH and AE inclined to each other at angle 100°, then ∠HOE =

(i) 80° (ii) 100° (iii) 40° (iv) 140°

(c) If ∠GOF = (3x – 8)° and ∠GCF = (2x + 3)° then x =

(i) 80° (ii) 100° (iii) 37° (iv) 73°

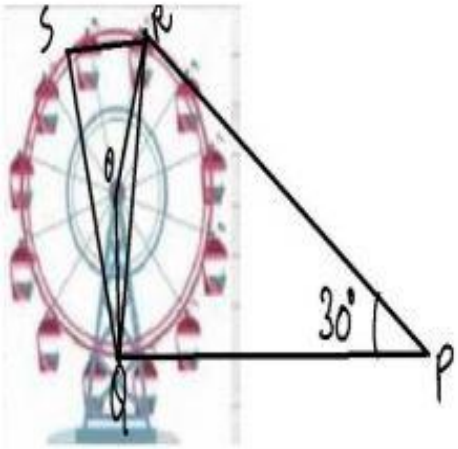
(d) DOHA is an/ a
(i) right angled triangle (ii) equilateral triangle (iii) both (i) and (ii) (iv) None of these

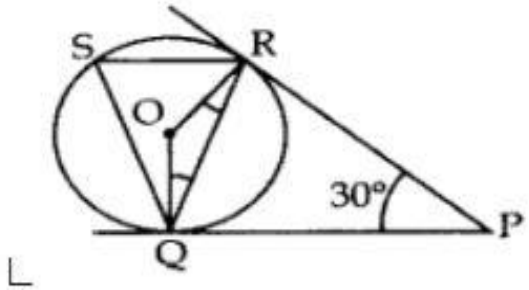
(e) ∠HAO =
(i) ∠HOE (ii) ∠AEO (iii) ∠AEB (iv) ∠OAE

CASE BASED QUESTIONS

26. A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride . She was curious about the different angles and measures that the wheel will form. She forms the figure as given below. **(AO2)**





(i) In the given figure find ∠ROQ

a) 60 b) 100 c) 150 d) 90

(ii) Find ∠RQP

a) 75 b) 60 c) 30 d) 90

(iii) Find ∠RSQ

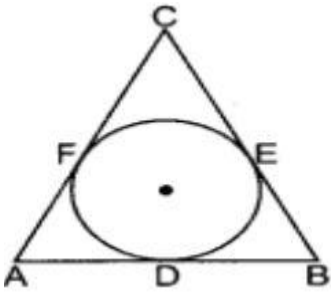
a) 60 b) 75 c) 100 d) 30

(iv) Find ∠ORP

a) 90 b) 70 c) 100 d) 60

27.

Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff . The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively. **(AO2)**



(i). Find the length of AD

(a) 7 b) 8 c) 5 d) 9

(ii). Find the Length of BE

(a) 8 b) 5 c) 2 d) 9

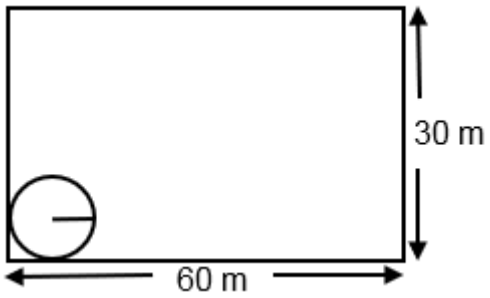
(iii). Find the length of CF

(a) 9 b) 5 c) 2 d) 3

(iv). If radius of the circle is 4cm, Find the area of ΔOAB

(a) 20 b) 36 c) 24 d) 48

28. A farmer has a rectangular field of length 60 m and breadth 30 m. By the farmer a pit of diameter 14 m is dug 12m deep to make a well. The earth taken out is spread in the field. Based on the above information, answer the following questions. **(AO2)**



(i) Find the volume of the earth taken out.
(a) 1460m^3 (b) 1462 m^3 (c) 1848 m^3 (d) 1850 m^3

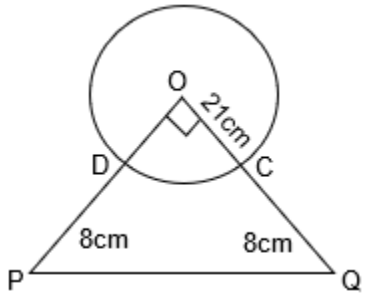
(ii) The area of the rectangular field is
(a) 1420 m^2 (b) 1430 m^2 (c) 1840 m^2 (d) 1800 m^2

(iii) Find the area of the top of the pit.
(a) 38.5 m^2 (b) 40.5 m^2 (c) 41.5 m^2 (d) None of these

(iv) The area of the remaining field is
(a) 1402.3 m^2 (b) 1184 m^2 (c) 1646 m^2 (d) 411.5 m^2

29. Committee of a club select a round glass trophy for awarding best director in film festival. Design of each trophy is made as shown in the figure, where its base PQRS is

golden plated from the front side and painted at the rate of 5 per cm^2 **(AO2)**



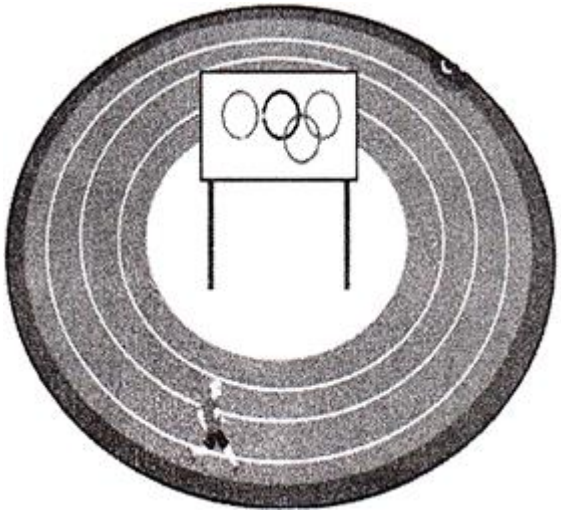
(i) Find the area of sector OSRO.
(a) 254 cm^2 (b) 348 cm^2 (c) 344.5 cm^2 (d) 346.5 cm^2

(ii) Find the area of ΔPOQ.
(a) 210 cm^2 (b) 420.5 cm^2 (c) 418.2 cm^2 (d) 300 cm^2

(iii) Find the total cost of golden plating.
(a) Rs. 276 (b) Rs. 280 (c) Rs. 284 (d) Rs. 370

(iv) Find the area of major sector formed in the given figure.
(a) 1400 cm^2 (b) 1039.5 cm^2 (c) 1462 cm^2 (d) 1472 cm^2

30. There is a race competition between all participants of a sports club, so that the sports committee can choose better person for a marathon. The race track in the club is in the form of a ring whose inner most circumference 176 m and the outer most circumference is 352 m. **(AO2)**



Based on the above information, answer the following questions.(i) Find the radius of the outer most circle.

(a) 48 m (b) 49 m (c) 50 m (d) 56 m

(ii) Find the radius of the inner most circle.

(a) 28 m (b) 40 m (c) 42m (d) 44m

(iii) Find the width of the track.

(a) 17 m (b) 28 m (c) 19 m (d) 10 m

(iv) Find the area of the race track.

(a) 7010 m^2 (b) 7392 m^2 (c) 8000 m^2 (d) 9002 m^2

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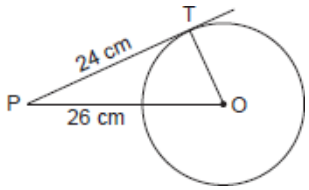
SOLUTIONS: CHAPTER 10-CIRCLES

MULTIPLE CHOICE QUESTIONS

1. (d) ∵ OT is radius and PT is tangent

∴ OT ⊥ PT Now, in ΔOTP,



$OP^2 = PT^2 + OT^2$

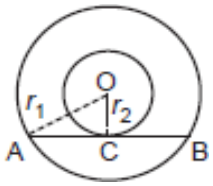
$\Rightarrow 26^2 = 24^2 + OT^2$

$\Rightarrow 676 - 576 = OT^2$

$\Rightarrow 100 = OT^2 \Rightarrow 10$ cm = OT

2. (c) normal

3. (c) ∵ AB touches



C(O, r2)

∴ OC ⊥ AB

Also, perpendicular from the centre to a chord bisects the chord.

∴ AC = BC

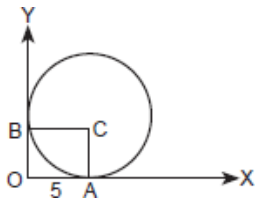
4. (c) Let radius of circle = R

∴ $\pi R2 = 25\pi$

⇒ R = 5 cm

∴ Distance between two parallel tangents = diameter = 2 × 5 = 10 cm.

5. (b) OA = OB ⇒ OB = 5



AC = BC [Radii]
⇒ OACB is a square.
⇒ AC = OA = 5
⇒ Diameter = 10 units.

6. (c) If we join OA the OAT is a right angled triangle
[since tangent is perpendicular to the circle]
now OT=4cm,∠OTA=30
therefore from cos rule we know AT=OTcos(30)= 4×√32= 2√3

7. (a) OP ⊥ PR [∵ Tangent and radius are ⊥ to each other at the point of contact]

∠ OPQ = 90º – 50º = 40º
OP = OQ [Radii]
∠ ∴∠ OPQ =
OQP = 40ºIn ΔOPQ,
∠ ⇒∠ POQ + OPQ + OQP = 180º
∠ ⇒ POQ + 40º +
40º = 180º

POQ = 180° – 80° = 100º.

8. (c) ∠AOB = 100°
∠OAB = ∠OBA
(∵ OA and OB are radii) Now, in ΔAOB,
∠AOB + ∠OAB + ∠OBA = 180°
(Angle sum property of Δ)
⇒ 100° + x + x = 180° [Let ∠OAB = ∠OBA = x]
⇒ 2x = 180° – 100°
⇒ 2x = 80° ⇒ x = 40° Also, ∠OAB + ∠BAT = 90°
[∵ OA is radius and TA is tangent at A]
⇒ 40° + ∠BAT = 90° ⇒ ∠BAT = 50°

ASSERTION AND REASON QUESTIONS

9. (c)

10. (a)

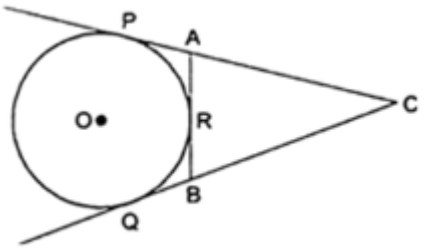
11. (b)

12. (a)

2 MARKS QUESTIONS

13: In ΔOAT, $\angle ATO = 40°$, $\angle OAT = 90°$
$\therefore$ $\angle AOT = 50°$ [Angle sum property]
Now $\angle BTO = 40°$ as OT bisects $\angle ATB$
Similarly, $\angle BOT = 50°$
$\therefore$ $\angle AOB = \angle AOT + \angle BOT = 50° + 50° = 100°$

14 :

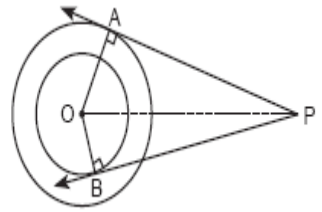


Since CP = CQ = 11cm [Length of the two tangents from same external point]
CQ = CB + BQ
But BQ = BR
Therefore, 11 = 7 + BR $\Rightarrow$ BR = 4 cm

15. AR = 4 cm
Also, AR = AQ $\Rightarrow$ AQ = 4 cm
Now, QC = AC – AQ = 11 cm – 4 cm = 7 cm ...(i)
Also, BP = BR
$\therefore$ BP = 3 cm and PC = QC
$\therefore$ PC = 7 cm [From (i)]
BC = BP + PC = 3 cm + 7 cm = 10 cm

16. PA = 12 cm, OA = 5 cm, OB = 3 cm
$OP^2 = OA^2 + AP^2 = OB^2 + BP^2$

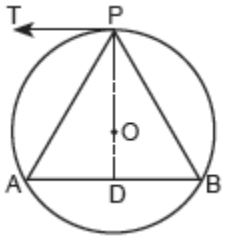
$\Rightarrow 25 + 144 = 9 + BP^2$
$\Rightarrow 169 - 9 = BP^2 \Rightarrow BP =$ cm = 12.65 cm. (Approx.)



17. If a circle touches all the four sides of quadrilateral ABCD, then
AB + CD = AD + BC
$\therefore$ 6 + 8 = AD + 9
$\Rightarrow$ 14 = AD + 9
$\Rightarrow$ 14 – 9 = AD
$\Rightarrow$ AD = 5 cm

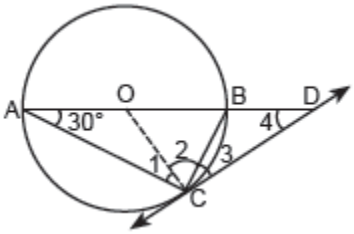
3 MARKS QUESTIONS

18. Join PO and produce it to D.
Here OP $\perp$ TP
$\Rightarrow$ $\angle OPT = 90°$
Also TP || AB
$\therefore$ $\angle TPD + \angle ADP = 180°$
$\Rightarrow$ $\angle ADP = 90°$
$\Rightarrow$ OD bisects AB [Perpendicular from the centre bisects the chord]
In ΔADP and ΔBDP
AD = BD [Proved]; $\angle ADP = \angle BDP$ [Each 90°]; PD = PD
$\therefore$ $\Delta ADP \cong \Delta BDP$ [SAS]
$\angle PAB = \angle PBA$ [C.P.C.T.]
$\therefore$ ΔPAB is isosceles triangle.

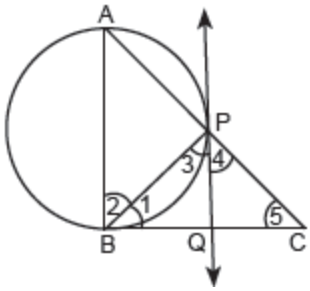


19. Join OC
OC $\perp$ CD $\therefore$ $\angle 2 + \angle 3 = 90°$
OC = OA $\therefore$ $\angle 1 = 30°$
Now $\angle 1 + \angle 2 = 90°$ [Angles in a semicircle]

$\therefore\ \angle 2 = 90° - 30° = 60°$
$\Rightarrow\ \angle 3 = 30°$
In ΔACD, $\angle ACD + \angle CAD + \angle 4 = 180°$
$\Rightarrow\ 120° + 30° + \angle 4 = 180°\ \Rightarrow\ \angle 4 = 30°$
In ΔBCD, $\angle 3 = \angle 4$
$\therefore$ BC = BD.



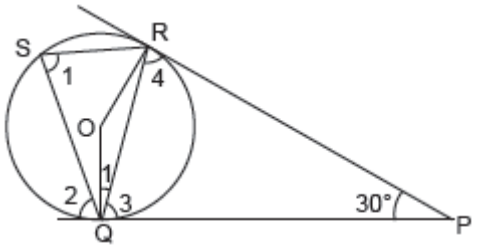
20. To prove: BQ = QC
AB is diameter
$\therefore\ \angle APB = 90°$ [Angle in semicircle]
$\Rightarrow$ BP $\perp$ AC
$\Rightarrow\ \angle 3 + \angle 4 = 90°$ ...(*i*)
BQ = QP [Tangents from Q] ...(*ii*)
$\therefore\ \angle 3 = \angle 1$
$\therefore$ Equation (*i*) becomes
$\Rightarrow\ \angle 1 + \angle 4 = 90°$
Also, $\angle 1 + \angle 5 = 90°$ [Sum of acute angles of right triangle]
$\therefore\ \angle 1 + \angle 4 = \angle 1 + \angle 5$
$\Rightarrow\ \angle 4 = \angle 5\ \Rightarrow$ PQ = QC ...(*iii*)
From (*ii*) and (*iii*) BQ = QC



21. BP = BQ (Tangents from B)
$\because$ BP = 27 cm,
$\therefore$ BQ = 27 cm and BC = 38 cm
Now, CQ = 38 – 27 = 11 cm;

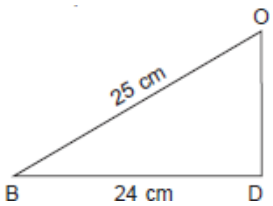
CQ = CR $\Rightarrow$ CR = 11 cm
$\because$ CD = DR + CR
$\Rightarrow$ 25 = DR + 11 $\Rightarrow$ DR = 14 cm ...(*i*)
Also, $\angle D = 90°$
$\because$ OR $\perp$ DC [Radius from point of contact of tangent]
$\therefore$ $\angle ORD = 90°$
Similarily $\angle OSD = 90°$
Also, RD = DS [Tangents from D]
$\Rightarrow$ OR = RD = 14 cm
$\therefore$ ORDS is a square.
$\therefore$ Radius of circle = 14 cm

22. In ΔRQP, QP = RP [Tangent from external point]
$\therefore$ $\angle 3 = \angle 4$
Now $\angle 3 + \angle 4 + 30° = 180°$
$\Rightarrow$ $2\angle 3 = 150°$ $\Rightarrow$ $\angle 3 = 75°$
Now $\angle QOR + \angle QPR = 180°$
$\Rightarrow$ $\angle QOR = 150°$
Now, $\angle 1 = \angle QOR$ $\Rightarrow$ $\angle 1 = 75°$
Also SR || QP
$\therefore$ $\angle 1 = \angle 2$ [Alternate interior angles]
$\Rightarrow$ $\angle 2 = 75°$
Now, $\angle 2 + \angle RQS + \angle 3 = 180°$
$\Rightarrow$ $\angle RQS = 180° - 150° = 30°$



<u>5 MARKS QUESTIONS</u>

23. (a) In ΔOBD, OD $\perp$ BD



(Radius is perpendicular to tangent)

$OB^2 = BD^2 + OD^2$

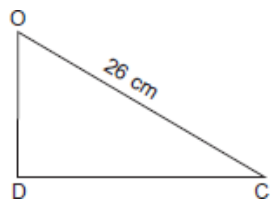
$\Rightarrow (25)^2 = (24)^2 + (OD)^2$

$\Rightarrow 625 = 576 + OD^2$

$\Rightarrow OD^2 = 625 – 576 = 49$

$\Rightarrow$ OD = 7 cm Radius of circle = 7 cm

(b) In ΔOCD, $OD \perp CD$



(Radius is perpendicular to tangent)

$\therefore\ OC^2 = OD^2 + CD^2$

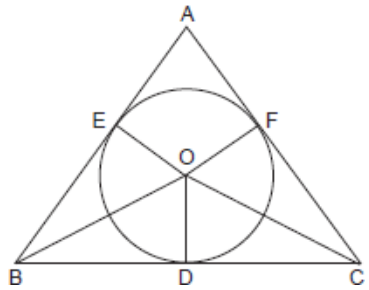
$\Rightarrow (26)^2 = (7)^2 + CD^2$

$\Rightarrow 676 = 49 + CD^2$

$\Rightarrow$ CD2 = 676 – 49 = 627 cm

$\Rightarrow$ CD = 25.04 cm

(c)



As BD = BE

CD = CF $\Rightarrow$ BF = 24 cm

CF = 25.0 cm

AE = AF = 8 cm

Perimeter of $\Delta ABC = AB + BC + AC$

$= (AE + BE) + (BD + CD) + (AF + FC)$

$= (8 + 24) + (24 + 25.04) + (8 + 25.04)$

$= 32 + 49.04 + 33.04 = 114.08$ cm

(d) Area of $\Delta BOC = \frac{1}{2} \times BC \times OD \times (49.04) \times 7$

$= 171.64$ cm^2

(e) Area of DABC = $\frac{1}{2}$ × (Perimeter of ΔABC) × radius of circle

$= \frac{1}{2} \times (114.08) \times 7 = 399.28$ cm^2

24) (a) 12 cm

(b) 10 cm

(c) A circle can have atomost two parallel tangents

(d) 60°

(e) No tangent can be drawn to a circle from a point lying inside the circle.

25. (a) (ii) 4 m

(b) (i) 80°

(c) (iii) x = 37°

(d) (i) right-angled triangle

(e) (iv) $\angle HAO = \angle OAE$

CASE BASED QUESTIONS

26 (i) (c) (ii) (a) (iii) (b) (iv) **(** a)

27. Since, tangent from same external points to the circle are equal in length .

Let,

- AD = AF = x cm .
- BD = BE = y cm .
- CF = CE = z cm .

so,

- AB = x + y = 12 cm .
- BC = y + z = 8 cm .
- CA = z + x = 10 cm .

adding all,

→ AB + BC + CA = 12 + 8 + 10

→ (x + y) + (y + z) + (z + x) = 30

→ 2(x + y + z) = 30

→ x + y + z = 15 cm .

then,

→ (x + y + z) - (y + z) = x => 15 - 8 = 7 cm = AD **(Ans. i)**

→ (x + y + z) - (x + z) = y => 15 - 10 = 5 cm = BE **(Ans. ii)**

→ (x + y + z) - (x + y) = z => 15 - 12 = 3 cm = CF **(Ans. iii)**

now, given that,

Radius of circle = OD = 4 cm.

Therefore,

→ Area ΔOAB = (1/2) * perpendicular height * Base = (1/2) * OD * AB = (1/2) * 4 * 12

= **24 cm² (Ans. iv)**

28 (i) (c) (ii)(d) (iii) (d) (iv) (c)

29) (i) (d) (ii) (b) (iii) (d) (iv) (b)

30) (i) (d) (ii) (a) (iii) (b) (iv) (b)

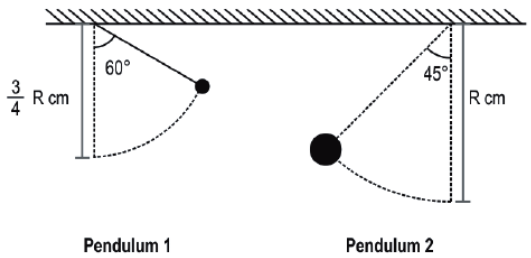
****

CHAPTER 11: AREAS RELATED TO CIRCLES

MULTIPLE CHOICE QUESTIONS

1. The area of a circular path of uniform width '*d*" surrounding a circular region of radius "*r*" is (AO2)
   (a) $\pi\, d(2r + d)$ (b) $\pi\,(2r + d)r$ (c) $\pi\,(d + r)r$ (d) $\pi\,(d + r)\,d$

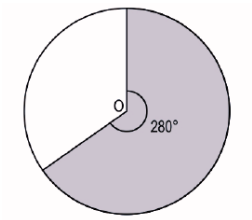
2. Shown below are two pendulums of different lengths attached to a bar.



   Based on the figure shown above the arc length of the pendulum 1 is_____ the arc length of pendulum 2. (AO2)

(a) greater than (b) lesser than (c) equal to (d) cannot be answered without knowing the value of R

3. Shown below is a circle with centre O.The shaded sector has an angle of 280^0 and area A cm^2. (AO2)



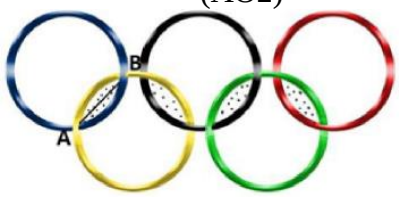
   Which of these is the area of the UNSHADED sector?
   (a) $\frac{2}{7} A\ cm^2$ (b) $\frac{1}{3} A\ cm^2$ (c) $\frac{2}{3} A\ cm^2$ (d) $\frac{7}{9} A\ cm^2$

4. If the perimeter of a semicircular protractor is 72 cm where $\pi = 22/7$, then the diameter of protractor is (AO1)
   (a) 14 cm (b) 33 cm (c) 28 cm (d) 42 cm

5. The area of a circle that can be inscribed in a square of side 10 cm is (AO1)
   (a) 40π cm² (b) 30π cm² (c) 100π cm² (d) 25π cm²

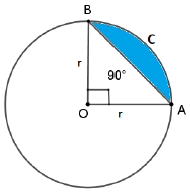
6. The perimeter of a square circumscribing a circle of radius '**a**'' units is (AO1)
   (a) 2a units (b) 4a units (c) 8a units (d) 16a units

7. Given below is the picture of the Olympic rings made by taking five congruent circles of radius 1cm each, intersecting in such a way that the chord formed by joining the point of intersection of two circles is also of length 1cm. Total area of all the dotted regions assuming the thickness of the rings to be negligible is (AO2)



(a) $4(\pi/12-\sqrt{3}/4)$ cm² (b) $(\pi/6 - \sqrt{3}/4)$ cm² (c) $4(\pi/6 - \sqrt{3}/4)$ cm² (d) $8(\pi/6 - \sqrt{3}/4)$ cm²

8. In the given figure, the area of the segment ACB is (AO2)



(a) $\frac{r^2}{4}(\pi-2)$ (b) $\frac{r^2}{4}(\pi+2)$ (c) $\frac{r^2}{4}(\pi-1)$ (d) $\frac{r^2}{4}(\pi+1)$

ASSERTION AND REASON QUESTIONS

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): In a circle of radius 6 cm , the angle of sector is 60° then the area of sector is $\frac{132}{7}$ cm^2 (AO1)

Reason (R): Area of a sector of a circle with radius r is $\frac{\theta}{360}\pi r^2$

10. Assertion (A): If the circumference of a circle is 176 cm, then its radius is 28 cm. (AO1)
Reason (R): Circumference = $2\pi \times$ radius.

11. Assertion (A): If the outer and inner diameter of a circular path is 10 m and 6 m respectively, then area of the path is $16\pi m^2$ (AO2)
Reason (R): If R and r be the radius of outer and inner circular path respectively, then area of circular path $\pi(R^2–r^2)$.

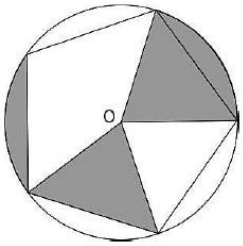
12. Assertion (A): The length of the minute hand of a clock is 7 cm. Then the area swept by the minute hand in 5 minute is 77/6 cm^2 (AO2)

Reason (R): The length of an arc of a sector of angle q and radius r is given by $l = \frac{\theta}{360} 2\Pi r$

2 MARKS QUESTIONS

13. Ronit drew two circles of different radius. Each circle had an arc which subtended equal angle at the centre. (AO2)
    Ronit said "both arcs are of same length".
    i) Is Ronit right?
    ii) If both radii and angles subtended by the two arcs are different, can the arc lengths be the same? Give reasons.

14. A regular pentagon is inscribed in a circle with centre O, of radius 5 cm, as shown below.



What is the area of the shaded part of the circle (find in terms of π)? (AO1)

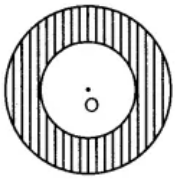
15. **Calculate the perimeter of an equilateral triangle if it inscribes a circle whose area is 154 cm^2.** (AO2)

16. If the perimeter of a circle is equal to that of a square, then find the ratio of their areas. (AO1)

17. Find the area of the square that can be inscribed in a circle of radius 8 cm. (AO2)

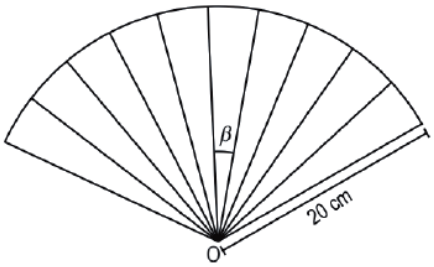
3 MARKS QUESTIONS

18. In the given figure, the area of the shaded region between two concentric circles is 286 cm^2. If the difference of the radii of the two circles is 7 cm, find the sum of their radii. (AO2)



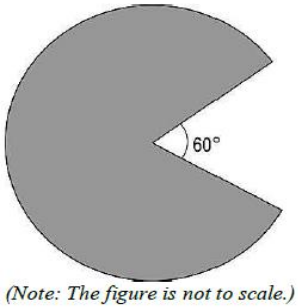
19. What is the angle subtended at the centre of a circle of radius 7 cm, by an arc of length $\frac{7}{4}$ π cm? (AO1)

20. The figure below is a part of a circle with centre O. Its area is $\frac{1250}{9}\pi\, cm^2$ and the 10 sectors are identical. Find the value of β. (AO2)



21. What is the perimeter of a quadrant of a circle (OAB) whose diameter is 10cm?
(Use π=3.14) . (AO1)

22. Wasim made a model of Pac-Man, after playing the famous video game of the same name. The area of the model is 120π cm^2. Pac-Man's mouth forms an angle of 60° at the centre of the circle.
A picture of the model is shown below. (AO1)

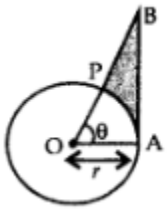


(Note: The figure is not to scale.)

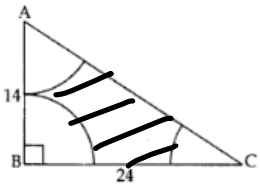
Wasim wants to decorate the model by attaching a coloured ribbon to the entire boundary of the shape. What is the minimum length of the ribbon required in terms of π? Show your work.

5 MARKS QUESTIONS

23. In Figure, is shown a sector OAP of a circle with centre 0, containing $\angle\theta$. AB is a perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\,[\tan\theta + \sec\theta + \pi\theta/180^\circ - 1]$ (AO2)

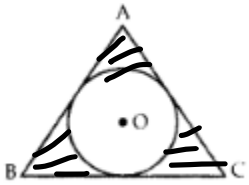


24. In Figure, ABC is a triangle right angled at B, with AB = 14 cm and BC = 24 cm. With the vertices A, B and C as centres, arcs are drawn, each of radius 7 cm. Find the area of the shaded region. (Use π = 22/7) (AO1)



25. In Figure, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of inscribed circle and the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

(AO2)



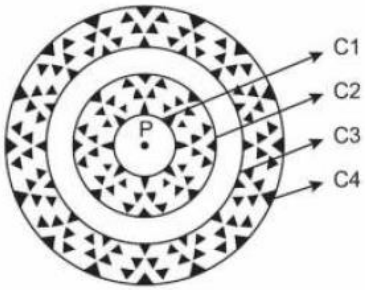
CASE BASED QUESTIONS (4 MARKS EACH)

26..Kritika bought a pendulum clock for her living room. The clock contains a small pendulum of length 15cm The minute hand and hour hand of the clock are 9 cm and 6 cm long respectively

(AO1)

i) Find the area swept by the minute hand in 10 minutes.
ii) Find the angles described by hour hand in 10 minutes.
iii) Find the area swept by hour hand between 11am and 5.00 pm.

27. Jaya drew this rangoli design during a competition. Circles C1, C2, C3 and C4 have common centre P.

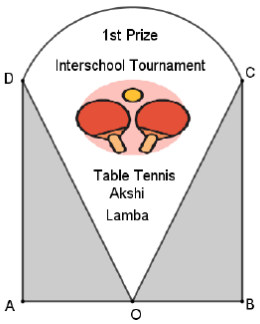


The table given below shows the radii of circles in terms of the radius of circle C1.The radius of circle C1 is 6 cm.

Radius of circle	Times of the radius of C1
C2	2
C3	2.5
C4	3.5

a) The area enclosed between circle C1 and C2 and circle C3 and C4 is painted. Find the area painted(in terms of π).(2 marks) (AO2)
b) Jaya wants to outline the boundary of circles C2 and C3 with ribbon.One roll of ribbon is 20cm long. How many rolls are required? (2marks) (AO1)

28. Shown below is the trophy shield Akshi received on winning an international Table tennis tournament.
The trophy is made of a glass sector DOC supported by identical wooden right triangles Δ DAO and Δ COB. Also, AO = 7 cm and AO**:** DA = 1**:** $\sqrt{3}$ (Use $\sqrt{3}$=1.73) (AO2)



Based on the given information, answer the following questions:

(i)Find $\angle DOC$ (1mark)

(ii)Find the area of the wooden triangles (1 mark)

(iii)Find the area of the shape formed by the glass portion.(2marks)

(OR)

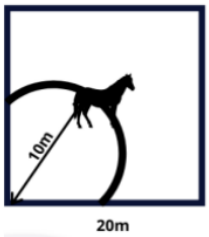
If Akshi wants to decorate the boundary of the glass portion with glitter tape, then find the length of the tape she needs.

29. Riya wants to bake a cake and divide it equally among 12 people as shown in the image above. The diameter of the cake is 30cm. (AO1)

(a) What is the total area of the top surface of the cake?(2marks)

(b) What is the area of each piece (top surface)? (2marks)

30. A horse is tied to a rope at one corner of a square grass field, of side 20m. The length of the rope with which it is tied is 10m. Using this information, answer the questions below: (AO1)



(a) Find the area of that part of the field in which the horse can graze.(2marks)

(b) Find the area of the part of the field that is left ungrazed by the horse.(2marks)

****

SOLUTIONS: CHAPTER 11- AREAS RELATED TO CIRCLES

MULTIPLE CHOICE QUESTIONS

1. (a) $\pi\, d(2r + d)$

2. (c) equal to

3. (a) $\frac{2}{7} A\, cm^2$

4. (c) 28 cm

5. (d) 25π cm²

6. (c) 8a units

7. (d) $8(\pi/6 - \sqrt{3}/4)$ cm²

8. (a) $\frac{r^2}{4}(\pi - 2)$

ASSERTION AND REASON QUESTIONS

9. option A

10. option A

11. option A

12. option B

2 MARKS QUESTIONS

13. (a) No,he is not right. Arc length is proportional to radius,here,radii are different.

(b)Yes,possible ,if the product of angle and radius are equal in both circles.

14. $10\pi cm^2$

15. $\pi \times r^2 = 154$ Or, r = 7 cm

Now, assume the length of each arm of the equilateral triangle to be "x" cm

So, the semi-perimeter of the equilateral triangle = (3x/2) cm

And the area of the equilateral triangle = $(\sqrt{3}/4) \times x^2$

We know, r = Area of triangle/semi-perimeter

So, $r = [x^2(\sqrt{3}/4)/ (3x/2)]$

=> $7 = \sqrt{3}x/6$

Or $x = 42/\sqrt{3}$

Multiply both numerator and denominator by $\sqrt{3}$

So, $x = 42\sqrt{3}/3 = 14\sqrt{3}$ cm

Now, the perimeter of an equilateral triangle will be = $3x = 3 \times 14\sqrt{3} = 72.7$ cm.

16.14:11

17. Diagonal of square $a\sqrt{2} = 16$cm

Finding $a=8\sqrt{2}$cm

Area = 128 cm^2

<u>3 MARKS QUESTIONS</u>

18. Equating area of shaded region

$\pi(R^2-r^2)=286$ cm^2

And finding R=10cm and r=3cm

19. Equating length of arc $= \frac{7}{4}\pi$ cm

Finding angle $=45^0$

20. Equates $\frac{1250}{9}\pi\, cm^2 = \frac{\theta}{360}\pi \times 20^2$

Finding $\Theta = 125^0$

$\beta = 12.5^0$

21. $\pi r/2 + r + r = 17.85$ cm^2

22. Equates $120\pi\, cm^2 = \frac{300}{360}\pi \times r^2$

R=12cm

Length of ribbon=20 π +24

5 MARKS QUESTIONS

23.

Length of arc AP (minor) $= \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{\theta}{180^\circ} \times \pi r$ ...(i)

$\angle OAB = 90^\circ$

In rt. ΔOAB, $\tan\theta = \frac{AB}{OA}$

$AB = OA \cdot \tan\theta$

$AB = r \cdot \tan\theta$ ...(ii)

In rt. ΔOAB, $\sec\theta = \frac{OB}{OA}$

$OB = OA \cdot \sec\theta$

$OB = r \cdot \sec\theta$

$\therefore \quad PB = OB - OP$

$= r\sec\theta - r$ ...(iii)

The perimeter of shaded region

$= AP + AB + PB$

$= \frac{\theta}{180^\circ}\pi r + r\tan\theta + r\sec\theta - r$

...[From (i), (ii)&(iii)

$= r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right)$ **(Hence proved)**

24.

Let $\angle BAC = \theta_1$, $\angle ABC = \theta_2$ and $\angle ACB = \theta_3$

Area of the shaded region

= ar(ΔABC) − [ar(sector A) + ar(sector B) + ar(sector C)]

$= \frac{1}{2} \times AB \times BC - \left[\frac{\theta_1}{360}\pi r^2 + \frac{\theta_2}{360}\pi r^2 + \frac{\theta_3}{360}\pi r^2\right]$

$= \frac{1}{2} \times 14 \times 24 - \frac{\pi r^2}{360}(\theta_1 + \theta_2 + \theta_3)$

$= 168 - \frac{1}{360} \times \frac{22}{7} \times 7 \times 7 \times 180$

...[$\because \theta_1 + \theta_2 + \theta_3 = 180$

$= 168 - 77 = \mathbf{91\ cm^2}$

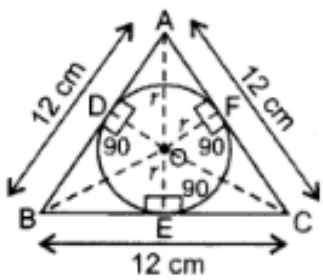
25.

Area of an Equilateral $\Delta = \frac{\sqrt{3}}{4}$ (side)^2

Construction: Draw OD ⊥ AB, OE ⊥ BC and OF ⊥ AC. Join OA, OB and OC.

Proof: Area of ΔABC

= ar(ΔAOB) + ar(ΔBOC) + ar(ΔAOC)



$$\Rightarrow \quad \frac{\sqrt{3}}{4}(12)^2 = \frac{AB \times r}{2} + \frac{BC \times r}{2} + \frac{AC \times r}{2}$$

$$\ldots\left[\text{ar. of equilateral } \Delta = \frac{\sqrt{3}}{4}a^2\right.$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4}(144) = \frac{r}{2}(AB + BC + AC)$$

$$\Rightarrow \quad 2 \times 36\sqrt{3} = r(12 + 12 + 12)$$

$$\Rightarrow \quad 72\sqrt{3} = 36r$$

$$\therefore \quad \textbf{Radius, } r = \left(\frac{72\sqrt{3}}{36}\right) = 2\sqrt{3} \text{ cm}$$

$$= 2(1.73) = \mathbf{3.46\ cm}$$

Area of the shaded region

= ar of ΔABC – ar of circle

$$= \frac{\sqrt{3}}{4}(12)^2 - 3.14 \times (2\sqrt{3})^2 \quad \ldots[\text{ar. of circle} = \pi r^2$$

$$= \frac{(1.73)}{4} \times 144 - 3.14 \times (12)$$

$$= 62.28 - 37.68 = \mathbf{24.6\ cm^2}$$

CASE BASED QUESTIONS

26. i) 42.42cm^2 ii) 5^0 iii) 56.568cm^2

27. 324π cm^2 and 9 rolls

28.

(i) Let $\angle DOA=\theta$, then

$\tan\theta=AD/AO=\sqrt{31}\Longrightarrow\theta=60°$

$\angle DOA=\angle COB=60°$

$\angle DOC=180°-(60°+60°)=60°$

(ii) Area of two wooden triangles

$=2\times1/2\times7\times7\sqrt{3}=84.77\ cm2$

(iii) $AO/DO=\cos60°\Longrightarrow7DO=1/2$

$\Longrightarrow DO=14\ cm$

Area of sector $DOC=60/360\times\pi\times14^2=102.67\ cm2$

OR

$AO/DO=\cos60°\Longrightarrow7DO=1/2$

$\Longrightarrow DO=14\ cm$

Length of tape required $=2\times14+60/360\times2\times\pi\times14=42.67\ cm$

29. 707.14cm^2 , 58.92 cm^2

30. 78.57 m^2 , 321.43m^2

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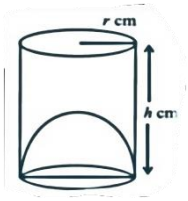
CHAPTER 12- SURFACE AREAS AND VOLUMES

MULTIPLE CHOICE QUESTIONS

1. If a cone is cut parallel to the base of it by a plane in two parts, then the shape of the top of the cone will be a:…… **(AO1)**

   a) Sphere b) Cube c) Cone d) Cylinder

2. The capacity of the cylindrical vessel with the hemispherical bottom portion raised upwards as shown in the given figure is …… (AO1)



   a) $\frac{\pi}{3}(3h - 2r)\, r^2$ b) $\frac{\pi}{3}(3h + r)\, r^2$

   c) $\frac{\pi}{2}(2h - 3r)\, r^2$ d) $\frac{\pi}{3}(2h + 3r)\, r^2$

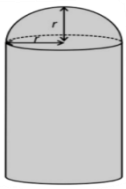
3. A tank is made of the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and radius is 30 cm. The total surface area of the tank is:…… **(AO2)**

   a) 30 m^2 b) 3.3 m^2 c) 30.3 m^2 d) 3300 m^2

4. If the diameter of the sphere is doubled, the surface area of the resultant becomes x times that of the original one. Then the value of x is …….. (AO1)

   a) 2 b) 3 c) 4 d) 5

5. Shown below is a solid made by joining right circular cylinder and a hemi sphere of equal radius 'r' cm. The total surface area of the solid is equal to the surface area of a sphere with twice the radius of this solid. **(AO2)**



   Which of the following gives the height of the cylinder in the above solid?

   a) 6r cm b) 6.5r cm c) 7 cm d) 7.5 cm

6. A boat which is in the shape of cuboid having a length 3m and breadth 2m is floating on a lake. The boat sinks by 1cm when a man gets on it. The mass of the man is ……. (AO2)

   a) 12 kg b) 60 kg c) 72 kg d) 96 kg

7. In a shower, 5cm of rain falls. The volume of water that falls on 1.5 hectares of ground is ……(AO2)

   a) 75cu. m b) 750cu.m c)7500cu.m d)75000cu.m

8. The radius of a wire is decreased to one third. If the volume remains the same, the length will become … (AO1)

   a) 3 times b) 6 times c) 9 times d) 27 times

ASSERTION AND REASON QUESTIONS

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

1. ASSERTION(A): If the height of the cone is 5 cm and diameter of the base is 24 cm, then the slant height of the cone is 25 cm.

REASON (R) : If 'r' be the radius and 'h' be the height of the cone, then slant height (l) $=\sqrt{r^2 + h^2}$ (AO1)

2. ASSERTION (A) : The total surface area of a cuboid is 2 (lb + bh + hl)

REASON (R) : Two identical solid cubes of side 5 cm are joined end to end, then the total surface of the resulting cuboid is 300 cm^2. (AO2)

3. ASSERTION (A): If diameter of a sphere is decreased by 25%, then its curved surface area is decreased by 43.75% (AO2)

REASON (R) : Curved surface area is increased when diameter decreases.

4. ASSERTION (A): Two cubes each of volume 125 cm^3 are re-joined end to end to form a cuboid, the surface area of the resulting cuboid is 250cm^2.

REASON(R) : If n cubes each of volume a^3 cubic units are joined end to end to form a cuboid. Then, the surface area of the resulting cuboid is 2 (2n + 1) a^2 sq. units (AO2)

2 MARKS QUESTIONS

1. A rectangular sheet of paper is 22 cm long and 12 cm wide. A cylinder is formed by rolling the paper along its length. Find the volume of cylinder. (AO2)

2. A cylindrical roller is used to paint a rectangular dais. The length of the roller is 1.4 m and its diameter is 0.7m. If the roller rolls over the dais 50 times completely, what is the area of the Dais? (AO2)

3. The sum of radius of base and height of a right circular cylinder is 37cm. If the total surface

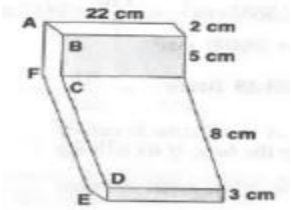
area is1628 cm^2, find the volume of cylinder. [Take $\pi = 22/7$]

4. A semi-circular sheet of metal of diameter 28cm is bent into an open conical cup. Find the depth and capacity of cup. (AO2)

5. A cone of height 24 cm and radius of base 6 cm is made up of modelling clay A child reshapes it in the form of a sphere Find the radius of the sphere? (AO2)

3 MARKS QUESTIONS

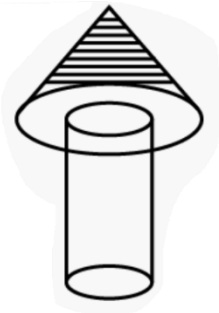
1. In the figure, the shape of a solid copper piece (made of two pieces) with dimensions as shown. The face ABCDEFA has uniform cross section. Assume that the angles at A, B, C, D, E and F are right angles. Calculate the volume of the piece. **(AO2)**



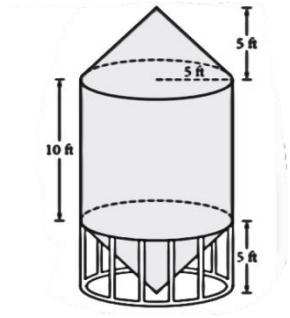
2. The radius and height of a solid right-circular cone are in the ratio of 5 : 12. If its volume is 314 cm^3, find the total surface area. [Take $\pi = 3.14$.] (AO2)

3. Find the volume and surface area of the largest cube that can be carved out of solid wooden sphere of radius $6\sqrt{3}$ cm (AO2)

4. A wooden toy is in the shape of a cone mounted on a cylinder as shown in the figure. If the height of the cone is 24 cm, the total height of the toy is 60 cm and the radius of the base of the cone is equal to 10 cm and its twice the radius of the base of the cylinder. Find the total surface area of the toy. [Take $\pi = 3.14$] (AO2)



5. A grain silo is built from two right circular cones and a right circular cylinder with internal measurements represented by the figure. Find the volume of the grain silo to the nearest cubic feet. (AO2)



5 - MARK QUESTIONS

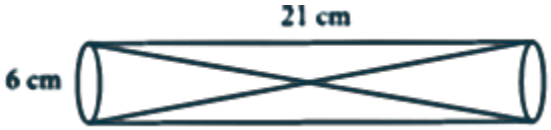
1. There are two identical solid cubical boxes of side 7cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find:

   i) the ratio of the total surface area of the two new solid formed. (AO2)

   ii) Volume of each new solid formed.

2. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

3. Two solid cones A and B are placed in a cylindrical tube as shown in the Figure. The ratio of their capacities are 2:1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.

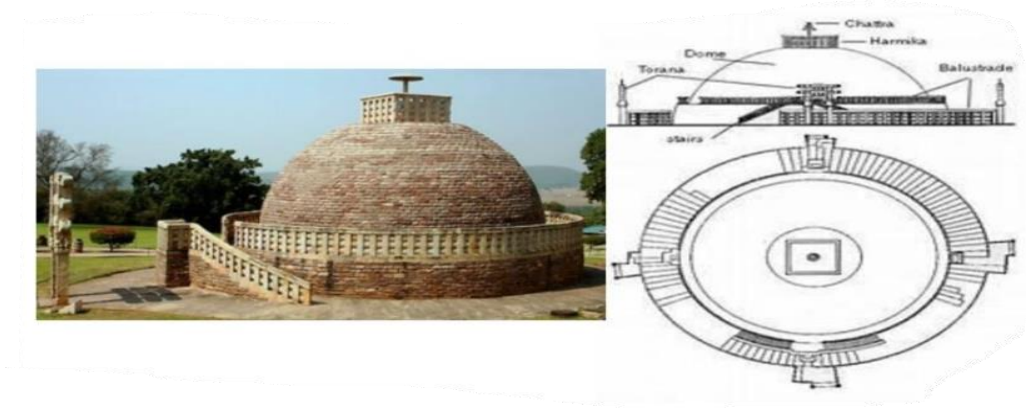


CASE BASED QUESTIONS

CASE STUDY-1

Read the passage and Answer the following questions.

The Great Stupa at Sanchi is one of the oldest stone structures in India, and an important monument of Indian Architecture. It was originally commissioned by the Emperor Ashoka in the 3rd century BCE. Its nucleus was a simple hemispherical brick structure built over the relics of the Buddha. It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it. (Take $\pi = 22/7$)



i). Calculate the volume of the hemispherical dome if the height of the dome is 21 m (AO2)

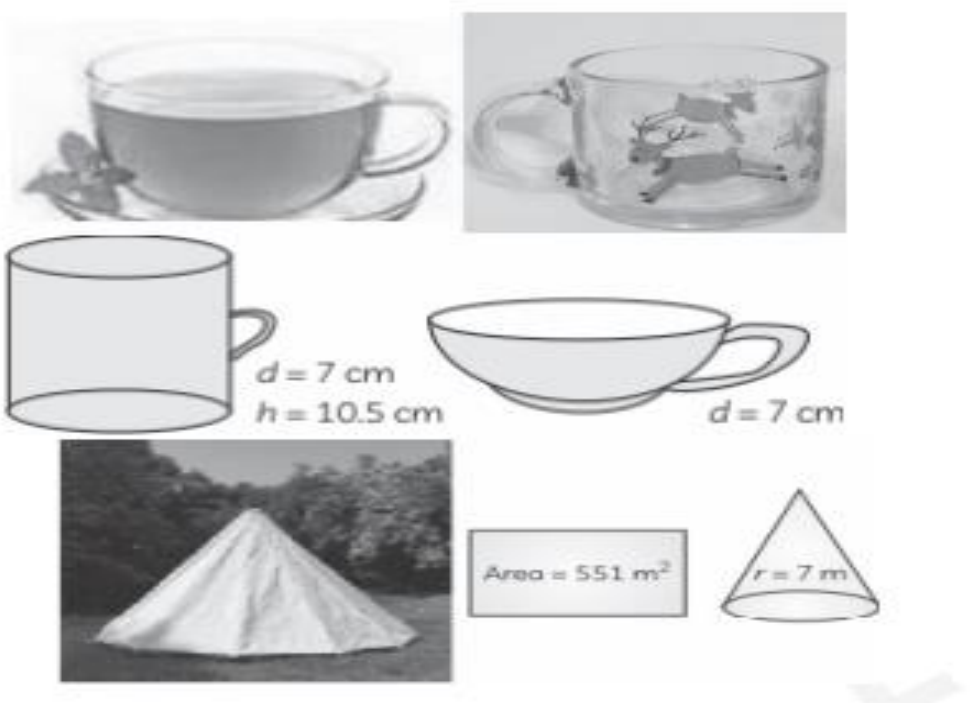
ii). How much cloth is required to cover the hemispherical dome if the radius of its base is 14m?(AO2)

iii). Find the total surface area of the combined figure i.e. hemispherical dome with radius 14m and cuboidal shaped top with dimensions are 8m , 6m and 4m. (AO2)

CASE STUDY-2

Read the passage and Answer the following questions.

Adventure camps are the perfect place for the children to practice decision making for them selves without parents and teachers guiding their every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass and some students in a hemispherical cup whose dimensions are shown below



After that they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas of area 551 m^2. Each group had to make a conical tent to accommodate all the four students. Assuming that all the wasting incurred while cutting and stitching, would amount to 1 m^2, the students put the tents. The radius of the tent is 7m

i) What is the volume of cylindrical cup? (AO2)

ii) What is the volume of hemispherical cup? (AO2)

iii) How much space on the ground is occupied by each student in the conical tent? (AO2)

CASE STUDY-3

Krishna nagar is a small town in Nadia District of West Bengal. Krishna nagar clay dolls are unique in their realism and quality of their finish. They are created by modelling coils of clay over a metal frame. The figures are painted in natural colours and their hair is made either by sheep's wool or jute. Artisans make models starting from fruits, animals, God, goddess, farmer, fisherman, weavers to Donald Duck and present comic characters. These creations are displayed in different national and international museums.

Here are a few images (not to scale) of some clay dolls of Krishna nagar

The ratio of diameters of red spherical apples in Doll-1 to that of spherical oranges in Doll-2 is 2:3. In Doll-3, male doll of blue colour has cylindrical body and a spherical head. The spherical head touches the cylindrical body. The radius of both the spherical head and the cylindrical body is 3cm and the height of the cylindrical body is 8cm.

Based on the above information answer the following questions:

i) What is the ratio of the surface areas of red spherical apples in Doll-1 to that of spherical oranges in Doll-2.? (AO1)

ii) The blue doll of Doll-3 is melted and its clay is used to make the cylindrical drum of Doll-4. If the radius of the drum is also 3cm, find the height of the drum. (AO2)

CASE STUDY-4

An ice –cream seller used to sell different kinds and different shapes of ice –cream like rectangular shaped, conical shape with one end hemispherical, rectangular shape with one end hemispherical and rectangular brick etc. One day sita came to his shop and purchased an ice cream which has the following shape: ice – cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm.

Answer the following questions:

1. Calculate the volume of only hemi spherical end of the ice cream (AO2)

2. Write the formula to calculate the total surface area of cone? (AO1)

3. What is the volume of the whole ice – cream? (AO2)

CASE STUDY-5

An antique box and its dimensions excluding the stand is given below.

i) Considering the thickness of the box to be negligible,

How much velvet cloth will be needed to cover the cuboidal inner area? (AO2)

ii) How many gold coins of diameter 2 cm and thickness 0. 5cm will fill $\frac{1^{th}}{7}$ of the volume of the dome of jewellery box. (AO2)

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SOLUTIONS: CHAPTER 12- SURFACE AREAS AND VOLUMES

MULTIPLE CHOICE QUESTIONS

1. c) Cone itself

2. a) $\frac{\pi}{3}(3h - 2r)\, r^2$

3. b) 3.3 m^2

4. c) 4

5. b) 6.5 cm

6. b) 60 kg

7. b) 750cu.m

8. c) 9 times

ASSERTION & REASONING QUESTIONS.

1. d) Assertion (A) is false but reason (R) is true

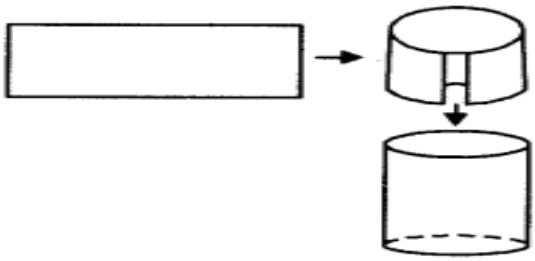
2. **c**) Assertion (A) is true but reason (R) is false

3. c) Assertion (A) is true but reason (R) is false.

4. a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A

2 MARKS QUESTIONS

1) As illustrated in figure length (22 cm) of paper will become circumference of the base of cylinder and breadth of paper will become height of cylinder.



$2\pi r = 22$ cm

$\Rightarrow r = 1\pi$ cm and $h = 12$ cm

Thus, volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (\frac{11}{\pi})^2 \times 12 = 462 \text{ cm}^3$

2) Length of roller = 1.4 m

Radius = diameter/2 = 0.7/2 = 0.35m

Surface Area of Roller = 2× 22/7 × 0.35 × 1.4= 3.08 sq m

Number of revolution = 50

Total area covered = 50 × 3.08 = 154 square meters

Area of Dais = 154 square meters

3) Sum of radius of base and height of a right circular cylinder = 37cm

$r + h = 37$

Total surface area = 1628cm^2

$2\pi r h + 2\pi r^2 = 1628$

$2\pi r (h + r) = 1628, \quad 2\pi r \times 37 = 1628$

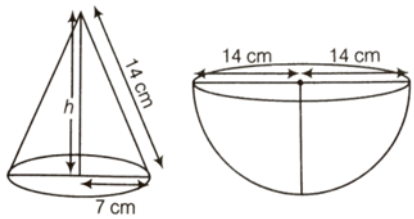
$r = 7$cm

Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$

4) Given, diameter of a semi-circular sheet = 28 cm

Radius of a semi-circular sheet (r) = 28/2 = 14cm

Since, a semi - circular sheet of metal is bent to from an open conical cup.



Let the radius of a conical cup = R

∴ Circumference of bases of cone = Circumference of semi- circle

$2\pi R = \pi r$

$\Rightarrow \quad 2\pi R = \pi \times 14 \quad \Rightarrow R = 7cm$

$h = \sqrt{l^2 - R^2} = \sqrt{14^2 - 7^2} = \sqrt{196 - 49} = \sqrt{147} = 12.1243cm$

Volume (capacity) of conical cup $= \frac{1}{3}\pi R^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12.1243 = 622.38\ cm^3$

Hence, the capacity of an open conical cup = 622.38 cm^3

5) cone of height (h) = 24 cm and radius of cone = 6 cm

Radius of sphere = R cm

Volume of Cone = Volume of sphere

$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$

R = 6 cm

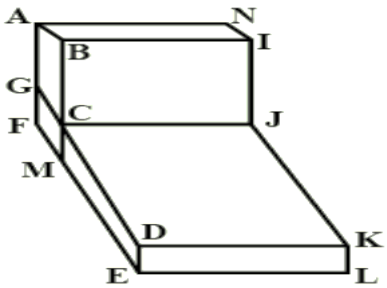
3 MARKS QUESTIONS

1) AB=2 cm

IJ=5 cm

JK=8 cm

KL=3 cm



Volume of upper cuboid = l × b × h

$= 22 \times 2 \times 5 = 220\ cm^3$

Volume of lower cuboid = FE × EL × DE = (8 + 2) × 22 × 3 = 660 cm^3

Volume of copper piece = (660 + 220) cm^3 = 880 cm^3

2) The ratio of radius and height of a solid right-circular cone = 5:12.

Let radius(r) = 5x and height (h) = 12x.

Volume = 314 cm^3.

$$\frac{1}{3}\pi r^2 h = 314$$

$$x^3 = \frac{314 \, x \, 3}{3.14 \, x \, 5x5x12}$$

x = 1 cm

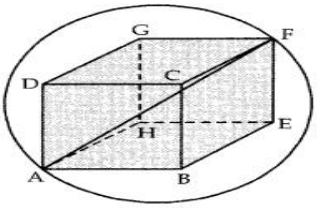
So, radius r = 5 cm and height h = 12 cm.

Using Pythagoras Theorem, slant height (l) = $\sqrt{h^2 + r^2}$

l = 13 cm

Total Surface Area of Cone = πr(r + l) = 3.14 x 5 x (5 + 13) = 3.14 x 5 x 18 = 282.6 cm^2

3)



For the largest cube (say ABCDEFGH) the diagonal of the cube (AF) must be along the diameter of the given sphere.

Length of diagonal of cube (AF) = 2r = 2 × 6√3 = 12√3

If edge of the cube is a cm, then diagonal AF= √3a

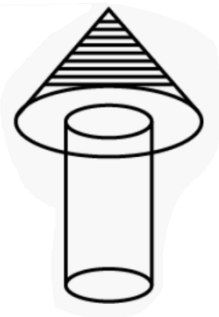
⇒ √3a = 12√3 ⇒ a = 12

∴ Volume of required cube = $a^3 = (12)^3$ = 1728 cm^3

Surface area of cube = $6a^2 = 6 \times (12)^2$ = 864 cm^2

4) Given: Height of cylinder (h) = Total height of toy - Height of cone = 60 −24 = 36 cm

Radius of cone(r) =10 cm



Slant height(l) = $\sqrt{10^2 + 24^2}$ = 26 cm

Surface Area of cone $=\pi r^2 + \pi rl = \pi(10^2+10\times26)$ cm^2

$=360\pi$ cm^2

According to given condition in the question:

Radius of cylinderR = 5 cm…(10^2 = R) or (r = 2R)

Surface Area of cylinder = $\pi r^2 + 2\pi rh = \pi(5^2 + 2\times5\times36)$ cm^2 $=385\pi$cm^2

Total Surface area of the toy = $(360+385)\pi$ cm^2 = = 2340.5 cm^2

5) The volume of the grain silo can be found by adding the volumes of all the solids of which it is composed.

The silo is made up of a cylinder with height = 10 feet and base radius (r) = 5 feet and two cones, each having height = 5 feet and base radius = 5 feet.

volume of cylinder (πr^2h) + volume of cone ($1/3\pi r^2h$) = Total volume of the silo.

Since the two cones have identical dimensions, the total volume, in cubic feet, of the silo is given by:

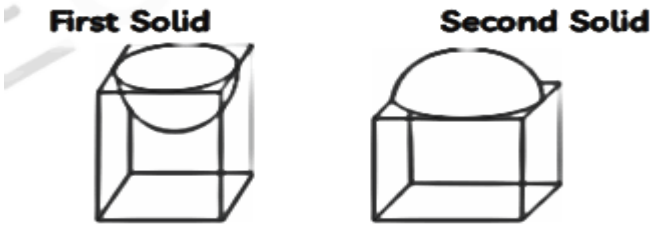
$$V = \pi(5)^2(10) + (2)(1/3)\,\pi\,(5)^2\,(5)$$

$$= (4/3)\,(250)\,\pi$$

$$=1047.2 \text{ cubic feet.(approx)}$$

<u>5 MARK QUESTIONS</u>

1)



i) SA for first new solid (S1):

$$6 \times 7 \times 7 + 2\pi \times (3.5)^2 - \pi \times 3.5^2$$

$$= 294 + 77 - 38.5 = 332.5 \text{ cm}^2$$

SA for second new solid (S2):

$$6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2$$

$$= 294 + 77 - 38.5 = 332.5 \text{ cm}^2$$

So S1 : S2 = 1:1

(ii) Volume for first new solid (V_1) = $7 \text{ x } 7 \text{ x } 7 - \frac{2}{3}\pi \text{ x } 3.5^3$

$$= 343 - \frac{539}{6} = \frac{1519}{6} \text{ cm}^3$$

Volume for Second new solid (V_2) = $7 \text{ x } 7 \text{ x} 7 + \frac{2}{3}\pi \text{ x } 3.5^3$

$$= 343 + \frac{539}{6} = \frac{2597}{6} \text{ cm}^3$$

2) Canal is the shape of cuboid where Breadth = 6 m

Depth = 1.5m

and speed of water = 10 km/hr

Length of water moved in 60 minutes i.e. 1 hour = 10 km

Length of water moved in 30 minutes i.e. $\frac{1}{2}$ hour = 5 km = 5000m

Volume of water moved from canal in 30 minutes = Length x Breadth x Depth

$$= 5000 \text{ x } 6 \text{ x } 1.5 \text{ m}^3$$

Volume of flowing water in canal = volume of water in area irrigated

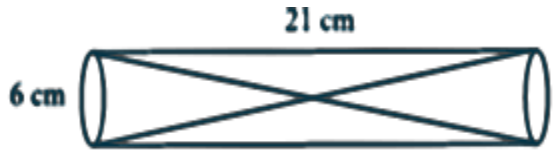
$$5000 \text{ x } 6 \text{ x } 1.5 \text{ m}^3 = \text{Area irrigated x } 8 \text{ cm}$$

$$5000 \text{ x } 6 \text{ x } 1.5 \text{ m}^3 = \text{Area irrigated x } \frac{8}{100} \text{ m}$$

$$\text{Area Irrigated} = \frac{5000 \, x \, 6 \, x \, 1.5 \, x \, 100}{8} \text{ m}^2$$

$= 5.625 \times 10^5 \text{ m}^2$

3)



Height of the tube = 21 cm

Base radius of the tube = 3cm

Volume of tube = $\pi r^2 h = \frac{22}{7} \times 3 \times 3 \times 21 = 594 \text{ cm}^3$

Let the height of cone A = h cm

Height of cone B = 21 – h cm

Base radius of both A and B = 3 cm

Volume of cone A = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 3^2 \times h = 3\pi h$

Volume of cone B = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 3^2 (21 - h)$

$= 3\pi (21 - h)$

It is given that the ratio of the volume = 2: 1

$\frac{3\pi h}{3\pi(21-h)} = \frac{2}{1}$

$h = 2(21 - h)$

$h = 42 - 2h$, $3h = 42$, $h = 14$

Height of cone A = 14 cm

Height of cone B = 21 – 4 = 7 cm

Volume of cone A = $3\pi h = 3 \times 3.14 \times 14 = 131.88 \text{ cm}^3$

Volume of cone B = $3\pi (21 - h) = 2 \times 3.14 \times 7 = 65.94 \text{ cm}^3$

The volume of remaining portion = Volume of the tube – the volume of cone A–the volume of cone B

$= 594 - 131.88 - 65.94 = 396.18 \text{ cm}^2$

Volume of remaining portion = 396.18 cm^3

CASE BASED QUESTIONS

CASE STUDY-1

i) The volume of hemispherical dome $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (21)^3$

= 19404 cu. m.

ii) The cloth required to cover the hemispherical dome = curved surface area of hemispherical dome.

r = 14 m

$= 2\pi r^2 = 2 \times \frac{22}{7} \times 14 \times 14$

$= 88 \times 14 = 1232 \text{ m}^2$

iii). Total surface area of the combined figure = surface area of hemispherical base + surface area of cuboidal top

$= 2\pi r^2 + 2hl + 2bh + lb$

$= 1232 + 2 \times 8 \times 4 + 2 \times 6 \times 4 + 6 \times 8$

$= 1232 + 64 + 48 + 48 = 1392\ m^2$

Surface area of hemispherical dome = $1392\ m^2$

CASE STUDY-2

i) Volume of cylindrical cup = $\pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10.5 = 404.25\ cm^3$

ii) Volume of cylindrical cup = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{11 \times 49}{6} = 89.83\ cm^3$

iii) Given that the canvas area = $551\ m^2$

Wastage of canvas = $1\ m^2$

Net canvas used for making the tent = original area of canvas – wastage = $551 - 1 = 550\ m^2$

Since this canvas is used to make the conical tent,

The curved surface area of a conical tent is = $\pi r l = \frac{22}{7} \times 7 \times l$

$550 = \frac{22}{7} \times 7 \times l$, $l = 25$ m,

$h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2} = \sqrt{576} = 24$ m

Ground space by each student in conical tent:

Ground space of the conical tent = area of the base of tent

$= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ m^2$

Since the tent can contain 4 students, hence the ground space will be equally shared by the 4 students.

Space covered by each student = $\frac{154}{4} = 38.5\ cm^2$

CASE STUDY-3

i) Let r_1 and r_2 be the radii of apples and oranges

$2r_1 : 2r_2 = 2 : 3$

The ratio of the surface areas of red spherical apples in Doll-1 to that of spherical oranges in Doll-2

$= 4\pi r_1^2 : 4\pi r_2^2 = 2^2 : 3^2$

$= 4 : 9$

ii) Let the height of the drum = h

Volume of the drum = volume of the cylinder + volume of the sphere

$\pi \times 3^2 \times h = (\pi \times 3^2 \times 8 + \frac{4}{3}\pi 3^3)$ cm^3

h = (8 + 4) cm

$\Rightarrow$ h = 12

height of the drum = 12 cm

CASE STUDY-4

i) Volume of only hemi spherical end of the ice cream = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 2.5 = \frac{1375}{42}$ = cm^3

ii) TSA of Cone = CSA of cone + base area = $\pi r l + \pi r^2$

iii) Volume of the whole ice – cream = Volume of cone + Volume of hemi sphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9 + \frac{2}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 2.5$

$= \frac{825}{14} + \frac{1375}{42} = \frac{275}{3} = 91.66$ cm^3

CASE STUDY-5

i) Velvet cloth will be needed to cover the cuboidal inner area = 2h (l + b) + lb

= 2 x 10 [30 + 14] + 30 x 14

= 20 x 44 + 420

= 880 + 420 = 1300 cm^2

ii) Diameter of coin = 2cm. radius (r) = 1 cm, h = 0.5 cm, R = 7 cm, H= 30cm

No. of coins $= \frac{\frac{1}{7}(volume\ of\ dome)}{volume\ of\ the\ coin} = \frac{\frac{1}{7} x \frac{1}{2} x \pi x R^2 x H}{\pi x r^2 x h} = \frac{\pi x 7 x 7x\ 30}{14 x \pi x 1 x 1 x 0.5}$ = 210 coins

CHAPTER 13-STATISTICS

MULTIPLE CHOICE QUESTIONS

1. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

the sum of lower limits of the median class and modal class is ……. (AO1)

(a) 15 (b) 25 (c) 30 (d) 35

2. If the difference of Mode and Median of a data is 24, then the difference of median and mean is.. (AO1)

(a) 8 (b) 12 (c) 24 (d) 36

3. The mean and mode of a frequency distribution are 28 and 16 respectively. The median is… (AO2)

(a) 22 (b) 23.5 (c) 24 (d) 24.5

4. The runs scored by a batsman in 35 different matches are given below:

Runs Scored	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	5	7	4	8	8	3

The lower limit of the median class is ……. (AO1)

(a) 15 (b) 30 (c) 45 (d) 60

5. The median class of the following data is (AO1)

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	8	10	12	22	30	18

(a) 20 – 30 (b) 30 – 40 (c) 40 – 50 (d) 50 – 60

6. For the following distribution:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	3	12	27	57	75	80

the modal class is …….. (AO1)

(a) 10 – 20 (b) 20 – 30 (c) 30 – 40 (d) 50 – 60

7. For the following distribution:

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

the upper limit of the median class is…… (AO1)

(a) 18.5 (b) 20.5 (c) 25.5 (d) 17.5

8. If the mean of the following distribution is 2.6, then the value of y is… (AO2)

Variable (x)	1	2	3	4	5
Frequency	4	5	y	1	2

(a) 3 (b) 8 (c) 13 (d) 24

ASSERTION AND REASON QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

1.Assertion (A): The arithmetic mean of the following given frequency distribution table is 13.81.

Marks	2.5 – 5.5	5.5 – 8.5	8.5 – 11.5	11.5 – 14.5	14.5 – 17.5	17.5 – 20.5
No. of Students	7	10	15	20	25	30

Reason (R): Mean = ∑fx / ∑f

2. Assertion (A): If the value of mode and mean is 60 and 66 respectively, then the value of median is 64.

Reason (R): Median = (mode + 2 mean)/2

3. Assertion (A): If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30.

Reason (R): Median value, if n is odd. $=\left(\frac{n+1}{2}\right)^{\text{th}}$

4. Assertion (A) : Class width = upper class limit – lower class limit

Reason (R) : Class mark =(Upper Class Limit+ Lower Class Limit)/2

2 MARK QUESTIONS

1. Calculate mode of the following data: (AO2)

Marks	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
No. of Students	5	10	12	6	3

2. Calculate median marks of the following data: (AO2)

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	2	12	22	8	6

3. Calculate mode of the following data (AO2)

Marks	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
No. of Students	7	5	10	12	6

4. Find the mean of the following distribution (AO2)

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency	5	10	10	7	8

5. Find median of the following data (AO2)

Marks	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100–120	120 –140
No. of Students	6	8	10	12	6	5	3

3 MARK QUESTIONS

1.Daily wages of 110 workers, obtained in a survey, are tabulated below: (AO2)

***Daily Wages* (*in Rs.*)**	100-120	120-140	140-160	160-180	180-200	200-220	220-240
Number of Workers	10	15	20	22	18	12	13

Compute the mean daily wages and modal daily wages of these workers.(AO2)

2.The table below shows the salaries of 280 persons (AO2)

Salary (in thousand Rs.)	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of persons	49	133	63	15	6	7	4	2	1

Calculate the median salary of the data.

3. The arithmetic mean of the following frequency distribution is 50. Find the value of p. (AO2)

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
frequency	17	*p*	32	24	19

4. Find the value of p, if the mode of the following distribution is 48: (AO2)

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 - 70	70 – 80
Frequency	7	14	13	12	*p*	18	15	8

5.The mean of the following frequency distribution is 25.2. Find the missing frequency x. (AO2)

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	8	x	10	11	9

5 MARK QUESTIONS

1. The median of the following data is 868. Find the values of x and y, if the total frequency is 100 (AO2)

Class	**Frequency**
800 – 820	7
820 – 840	14
840 – 860	x
860 – 880	25
880 – 900	y
900 – 920	10
920 – 940	5

2. The distribution below gives the makes of 100 students of a class, if the median makes are 24, find the frequencies f_1 and f_2 (AO2)

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students	4	6	10	f_1	25	f_2	18	5

3.A survey regarding the heights (in cm) of 50 girls of class X^{th} of a school was conducted and the following data was obtained. Find the mean, median and mode of the given data. (AO2)

Heights (in cm)	120 – 130	130 – 140	140 – 150	150 – 160	160 – 170
No. of Girls	2	8	12	20	8

CASE STUDY BASED QUESTIONS

1. The COVID-19 pandemic, also known as the coronavirus pandemic, is an ongoing pandemic of Coronavirus disease 2019 (COVID-19) caused by severe acute respiratory syndrome coronavirus 2(SARS-CoV-2). It was first identified in December 2019 in Wuhan, China. During survey, the ages of 80 patients infected by COVID and admitted in the one of the City hospital were recorded and the collected data is represented in the less than cumulative frequency distribution table.

Age(in year)	Below 15	Below 25	Below 35	Below 45	Below 55	Below 65
No. of patients	6	17	38	61	75	80

Based on the above information, answer the following questions.

(i) Find the modal class interval. (AO1)

(ii) Find the median class interval. (AO1)

(iii) Find the modal age of the patients admitted in the hospital. (AO2)

OR

iv) Find the median age of the patients admitted in the hospital. (AO2)

2. Overweight and obesity may increase the risk of many health problems, including diabetes, heart disease, and certain cancers. The basic reason behind is the laziness, eating more junk foods and less physical exercise. The school management give instruction to the school to collect the weight data of each student.

During medical check of 35 students from Class X- A, there weight was recorded as follows:

Weight (in kg)	**No. of Students**
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

(i) Find the median class of the given data. (AO1)

(ii) Find the modal class of the given data. (AO1)

(iii) Calculate the median weight of the given data. (AO2)

OR

iv) Find the mean of the given data. (AO2)

3. A group of students decided to make a project on Statistics. They are collecting the heights (in cm) of their 51 girls of Class X-A, B and C of their school. After collecting the data, they arranged the data in the following less than cumulative frequency distribution table form:

Height (in cm)	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

i) What is the lower limit of median class? (AO1)

ii) What is the upper limit of modal class? (AO1)

iii) What is the mean of lower limits of median and modal class? (AO1)

OR

iv)What is of the median? (AO2)

4. A group of students went to another city to collect the data of monthly consumptions (in units) to complete their Statistics project. They prepare the following frequency distribution table from the collected data gives the monthly consumers of a locality.

Monthly consumption(in units)	No. of consumers
65 – 85	4
85 -105	5
105 - 125	13
125 – 145	20
145 - 165	14
165 - 185	8
185 - 205	4

i) What is the lower limit of median class? (AO1)

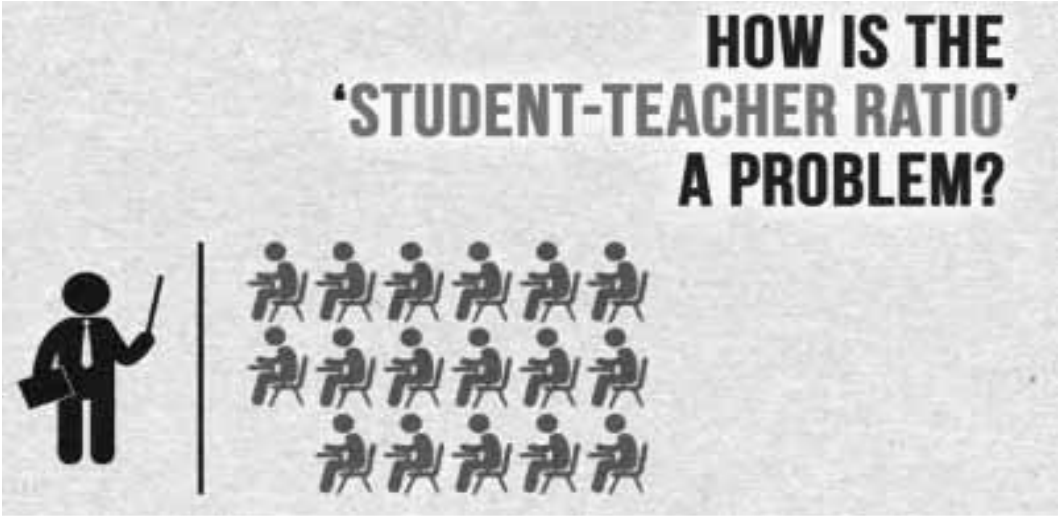
ii) What is the lower limit of modal class? (AO1)

iii) Find the median? (AO2)

(OR)

iv) Find the mean? (AO2)

5. Student-teacher ratio expresses the relationship between the number of students enrolled in a school and the number teachers employed by the school. Student-teacher ratio is important for a number of reasons. It can be used as a tool to measure teacher workload as well as the allocation of resources. A low student-teacher ratio indicates the burden on a single teacher of teaching multiple students as well as the lack of time that each student gets.



A survey was conducted in the 100 secondary school of Rajasthan and following frequency distribution table was prepared:

Students per teacher	Number of School
20 - 25	5
25 - 30	15
30 - 35	25
35 - 40	30
40 - 45	15
45 - 50	10

i) What is the upper limit of median class? (AO1)

ii) What is the lower limit of model class? (AO1)

iii) What is the median value of students per teacher? (AO2)

****

SOLUTIONS: CHAPTER 13-STATISTICS

MULTIPLE CHOICE QUESTIONS

1.

Ans: (b) 25

Class	Frequency (f)	c.f.
0 – 5	10	10
5 – 10	15	25
10 – 15	12	37
15 – 20	20	57
20 – 25	9	66
	$N = 66$	

Since, $N = 66$, then $\frac{N}{2} = 33$

and cumulative frequency greater than or equal to 33 lies in class 10 – 15

So, median class is 10 – 15

$\therefore$ Lower limit of median class is 10

and highest frequency is 20 lie in class 15 – 20

So, modal class is 15 – 20.

$\therefore$ Lower limit of modal class is 15.

Hence, sum of lower limits of the median and modal class is 10 + 15 = 25.

Ans: (b) 12

mode – median = 24 (given)

∴ mode = 24 + median

Since, mode = 3 median – 2 mean [By empirical relation]

∴ 24 + median = 3 median – 2 mean

⇒ 2 median – 2 mean = 24

⇒ median – mean = 12

3.

Ans: (c) 24

We know that, Mode = 3 Median – 2 Mean

⇒ 3 Median = Mode + 2 Mean

⇒ 3 Median = 16 + 2 × 28 ⇒ Median = 72/3 = 24

4

Ans: (c) 45

Runs Scored	0-15	15-30	30-45	45-60	60-75	75-90
Frequency	5	7	4	8	8	3
cf	5	12	16	24	32	35

Here, n = 35 ⇒ n/2 = 17.5

Median class is 45 – 60

Hence, lower limit is 45

5.

Ans: (b) 30 – 40

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	8	10	12	22	30	18
cf	8	18	30	52	82	100

Here, n=100, So $\frac{n}{2} = 50$

The cumulative frequency, just greater than 50, is 52 which belongs to class 30 – 40.

So, the median class is 30 – 40.

6

(a) 10 – 20 (b) 20 – 30 (c) 30 – 40 (d) 50 – 60

Ans: (c) 30 – 40

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students	3	9	15	30	18	5

Highest frequency is 30 which belong to 30 – 40. Hence, Modal class is 30 – 40

7.

Ans:

Class	**Frequency**	**Cf**
– 0.5 – 5.5	13	13
5.5 – 11.5	10	23
11.5 – 17.5	15	38
17.5 – 23.5	8	46
23.5 – 29.5	11	57

Here, n = 57 So, $\frac{n}{2}$ = 28.5

The cumulative frequency, just greater than 28.5, is 38 which belongs to class 11.5 – 17.5.
So, the median class is 11.5 – 17.5 Its upper limit is 17.5

8.

Ans: (b) 8

Variable (x)	1	2	3	4	5	**Total**
Frequency (f)	4	5	y	1	2	y + 12
fx	4	10	3y	4	10	3y + 28

Here, ∑f = y + 12 and ∑fx = 3y + 28

$$Mean, \bar{x} = \frac{\sum fx}{\sum f} \Rightarrow 2.6 = \frac{3y+28}{y+12} \Rightarrow 3y+28 = 2.6y+31.2$$

$$\Rightarrow 0.4y = 3.2 \Rightarrow y = 8$$

ASSERTION AND REASON QUESTIONS

1. Assertion (A): The arithmetic mean of the following given frequency distribution table is 13.81.

Marks	2.5 –5.5	5.5 –8.5	8.5 –11.5	11.5 –14.5	14.5 –17.5	17.5 -20.5
No. of Students	7	10	15	20	25	30

Reason (R): Mean = ∑fx / ∑f

Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Marks	2.5 – 5.5	5.5 – 8.5	8.5 – 11.5	11.5 – 14.5	14.5 – 17.5	17.5 – 20.5
No. of Students	7	10	15	20	25	30
Class mark 'x'	4	7	10	13	16	19
Fx	28	70	150	260	400	570

Here, ∑fx = 1478, ∑f = 107

Mean = ∑fx / ∑f = 1478 / 107 = 13.81

2. Assertion (A): If the value of mode and mean is 60 and 66 respectively, then the value of median is 64.

Reason (R): Median = (mode + 2 mean)/2

Ans: (c) Assertion (A) is true but reason (R) is false.

$$Median = \frac{1}{3}(\text{mod}\,e + 2mean) = \frac{1}{3}[60+2(66)] = \frac{1}{3}\times 192 = 64$$

3. Assertion (A): If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30.

Reason (R): Median value, if n is odd. $= \left(\frac{n+1}{2}\right)^{th}$

Answer: d

4. Assertion (A) : Class width = upper class limit – lower class limit

Reason (R) : Class mark =(Upper Class Limit+ Lower Class Limit)/2

Answer: b

2 MARKS QUESTIONS

1. Calculate mode of the following data:

Marks	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
No. of Students	5	10	12	6	3

Ans: Since the maximum frequency is 12 which belongs to 40 – 60, therefore modal class is 40 – 60
Here, $l = 40$, $f_0 = 10$, $f_1 = 12$, $f_2 = 6$, $h = 20$

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow Mode = 40 + \frac{12-10}{24-10-6} \times 20 = 40 + \frac{2}{8} \times 20 = 40 + 5 = 45$$

2. Calculate median marks of the following data:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	2	12	22	8	6

Ans:

Classes	Number of students	c.f.
0 – 10	2	2
10 – 20	12	14
20 – 30	22	36
30 – 40	8	44
40 – 50	6	50

$n = 50$, $\frac{n}{2} = \frac{50}{2} = 25$, Median Class = 20 – 30
$l = 20$, $f = 22$, $c.f. = 14$, $h = 10$

$$\text{Median} = l + \frac{\left(\frac{n}{2} - c.f.\right)}{f} \times h = 20 + \frac{(25-14)}{22} \times 10 = 20 + \frac{11}{22} \times 10 = 20 + 5 = 25$$

3. Calculate mode of the following data:

Marks	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
No. of Students	7	5	10	12	6

Ans:

Modal class = 18 – 24
$\therefore$ $p = 18$, $f_0 = 10$, $f_1 = 12$, $f_2 = 6$, $h = 6$

$$\therefore \text{Mode} = \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 18 + \frac{12-10}{24-10-6} \times 6 = 18 + \frac{12}{8} = 18 + 1.5 = 19.5$$

4. Find the mean of the following distribution:

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Frequency	5	10	10	7	8

Ans:

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13	**Total**
Frequency 'f'	5	10	10	7	8	40
Class mark 'x'	4	6	8	10	12	
fx	20	60	80	70	96	326

$$Mean, \bar{x} = \frac{\sum fx}{\sum f} = \frac{326}{40} = 8.15$$

5. Find median of the following data:

Marks	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100–120	120 –140
No. of Students	6	8	10	12	6	5	3
cf	6	14	24	36	42	47	50

Ans: Median = l + [n/2- cf]/f x h

= 40 + [25 – 24] /10 x 20 = 40 + 2 = 42

3 MARKS QUESTIONS

1. Daily wages of 110 workers, obtained in a survey, are tabulated below:

***Daily Wages* (*in Rs.*)**	100-120	120-140	140-160	160-180	180-200	200-220	220-240
Number of Workers	10	15	20	22	18	12	13

Compute the mean daily wages and modal daily wages of these workers.

Ans:

Daily Wages (in ₹)	*Number of Workers (f_i)*	x_i	u_i	$f_i u_i$
100–120	10	110	–3	–30
120–140	15	130	–2	–30
140–160	20	150	–1	–20
160–180	22	170	0	0
180–200	18	190	1	18
200–220	12	210	2	24
220–240	13	230	3	39
Total	110			1

Mean daily wages

$$= 170 + \frac{1}{110} \times 20$$

= ₹170.19 (approx.)

$$\text{Mode} = 160 + \frac{22 - 20}{44 - 20 - 18} \times 20$$

= ₹166.67 (approx.)

2.The table below shows the salaries of 280 persons:

Salary (in thousand Rs.)	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of persons	49	133	63	15	6	7	4	2	1

Calculate the median salary of the data.

Ans:

Salary (in thousand Rs.)	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of persons	49	133	63	15	6	7	4	2	1
cf	49	182	245	260	266	273	277	279	280

Here, n = 280 ⇒ n/2 = 140

⇒ Median class is 10-15
l = 10, cf = 49, f = 133, h = 5

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$Median = 10 + \left(\frac{140 - 49}{133} \right) \times 5 = 10 + \frac{91 \times 5}{133} = 10 + \frac{455}{133} = 10 + 3.421 = 13.42$$

Hence, median salary is Rs. 13.42 (in thousand)

3.The arithmetic mean of the following frequency distribution is 50. Find the value of p.

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
frequency	17	p	32	24	19

Ans:

Class	x_i	Frequency f_i	$f_i x_i$
0 – 20	10	17	170
20 – 40	30	p	$30p$
40 – 60	50	32	1600
60 – 80	70	24	1680
80 – 100	90	19	1710
Total		$\Sigma f_i = 92 + p$	$\Sigma f_i x_i = 5160 + 30p$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 50 = \frac{5160 + 30p}{92 + p}$$

$\Rightarrow$ 50 × 92 + 50p = 5160 + 30p
$\Rightarrow$ 50p – 30p = 5160 – 4600
$\Rightarrow$ 20p = 560 $\Rightarrow$ $p = \frac{560}{20} = 28$

4. Find the value of p, if the mode of the following distribution is 48:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 - 70	70 – 80
Frequency	7	14	13	12	p	18	15	8

Ans: mode = 48, model class = 40 – 50
f_1 = p, f_0 = 12, f_2 = 18 , l= 40 and h = 10
Using the mode formula, p = 20

5. The mean of the following frequency distribution is 25.2. Find the missing frequency x.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	8	x	10	11	9
xi	5	15	25	35	45
$f_i x_i$	40	15x	250	385	405

Mean = 25.2, $\sum f_i xi$ = 1080 + 15x, $\sum f_i$ = 38+ x
1080 + 15x = 957.6 + 25.2
x = 12

5 - MARK QUESTIONS

1.The median of the following data is 868. Find the values of x and y, if the total frequency is 100

Class	Frequency
800 – 820	7
820 – 840	14
840 – 860	x
860 – 880	25
880 – 900	y
900 – 920	10
920 – 940	5

Ans:

Class	Frequency	Frequency
800 – 820	7	7
820 – 840	14	21
840 – 860	x	x + 21
860 – 880	25	x + 46
880 – 900	y	x + y + 46
900 – 920	10	x + y + 56
920 – 940	5	x + y + 61

From table, we have x + y + 61 = 100 ⇒ x + y = 100 – 61 ⇒ x + y = 39
Here, median = 868, therefore median class is 860 – 880
So, l = 860, cf = x + 21, f = 25, h = 20, n/2 = 50

Now, $Median = l + \left(\frac{\frac{n}{2} - cf}{f} \times h \right) \Rightarrow 868 = 860 + \left(\frac{50 - (x+21)}{25} \times 20 \right)$

$\Rightarrow 868 - 860 = \left(\frac{50 - x - 21)}{5} \times 4 \right) \Rightarrow 8 = \frac{29 - x}{5} \times 4$

$\Rightarrow 40 = (29 - x)4 \Rightarrow 29 - x = 10 \Rightarrow x = 29 - 10 = 19$

$\Rightarrow y = 39 - 19 = 20$

2.The distribution below gives the makes of 100 students of a class, if the median makes are 24, find the frequencies f_1 and f_2

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students	4	6	10	f_1	25	f_2	18	5

Ans:

Class	Frequency	cf
0 – 5	4	4
5 – 10	6	10
10 – 15	10	20
15 – 20	f_1	20 + f_1
20 – 25	25	45 + f_1

25 – 30	f_2	$45 + f_1 + f_2$
30 – 35	18	$63 + f_1 + f_2$
35 – 40	5	$68 + f_1 + f_2$

Now, Median = 24 (Given)
So, median class = 20 – 25
For this class,
I = 20. h = 5, n/2 = 50, cf = 20 + f_1, f = 25

We know, Median = $l + \left(\dfrac{\frac{n}{2} - cf}{f} \times h\right)$

$$\Rightarrow 24 = 20 + \frac{50-(20+f_1)}{25} \times 5 \;\Rightarrow 4 = \frac{30-f_1}{5} \Rightarrow 30 - f_1 = 20 \Rightarrow f_1 = 10$$

Also, Sum of of frequencies=100
$\Rightarrow 68 + f_1 + f_2 = 100 \qquad \Rightarrow f_1 + f_2 = 32 \;\Rightarrow 10 + f_2 = 32 \Rightarrow f_2 = 22$
$\therefore f_1 = 10, f_2 = 22.$

3. A survey regarding the heights (in cm) of 50 girls of class Xth of a school was conducted and the following data was obtained. Find the mean, median and mode of the given data.

Heights(incm)	120–130	130–140	140 – 150	150 – 160	160 – 170
No. of Girls	2	8	12	20	8

Ans: Let the assumed mean (A) = 145.
Class interval (h) = 10.

Class	Frequency (fi)	Mid value(xi)	ui=((xi−A)/h)	fiui	C.F	
120-130	2	125	-2	-4	2	
130-140	8	135	-1	-4	10	
140-150	12	145 = A	0	0	22	
150-160	20	155	1	20	42	
160-170	8	165	2	16	50	
	N=50		∑(fiui)=24			

i) Mean
X = A+h(∑fiui/N)
= 145+10 x (24/50)
=145+4.8 = 149.8

ii) N = 50, N/2 = 25
Cumulative frequency just after 25 is 42
Median class is 150-160
I = 150, h = 10, N = 50, c = 22, f = 20
Median =I+h((N/2−c)/f)
=150+10(25−22/20)
= 30 + 10 x 3/20

= 150 + 1.5 = 151.5

iii) Mode = 3 x median - 2 x mean

= 3 x 151.5 - 2 x 149.8 = 454.5 - 299.6

= 154.9

Thus, mean = 149.8, median = 151.5, mode = 154.9

CASE STUDY BASED QUESTIONS

1) The COVID-19 pandemic, also known as the coronavirus pandemic, is an ongoing pandemic of coronavirus disease 2019 (COVID-19) caused by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). It was first identified in December 2019 in Wuhan, China.

During survey, the ages of 80 patients infected by COVID and admitted in the one of the City hospital were recorded and the collected data is represented in the less than cumulative frequency distribution table.

Age(in year)	Below 15	Below 25	Below 35	Below 45	Below 55	Below 65
No. of patients	6	17	38	61	75	80

Based on the above information, answer the following questions.

i) Find the modal class interval.

ii) Find the median class interval

iii) Find the modal age of the patients admitted in the hospital.

OR

iv) Find the median age of the patients admitted in the hospital.

Ans: i) Since the highest frequency is 23 which belongs to 35 – 45.

Therefore, modal class is 35 – 45.

Age(in yrs)	No. of patients	cf
5 – 15	6	6
15 – 25	11	17
25 – 35	21	38
35 – 45	23	61
45 – 55	14	75
55 – 65	5	80

ii) Here, n = 80 ⇒ n/2 = 80/2 = 40 which lies in 35 – 45

Therefore, medial class is 35 – 45.

iii) Here, l = 35, f_0 = 21, f_1= 23, f_2 = 14, h = 10

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow Mode = 35 + \frac{23-21}{46-21-14} \times 10 \ = 35 + \frac{2}{11} \times 10 = 36.8$$

OR

iv) Here, n = 80 ⇒ n/2 = 40, therefore median class is 35 – 45

So, l = 35, cf = 38, f = 23, h = 10

Now, $Median = l + \left(\frac{\frac{n}{2} - cf}{f} \times h \right) \Rightarrow Median = 35 + \left(\frac{40-38}{23} \times 10 \right)$

$$\Rightarrow Median = 35 + \left(\frac{20}{23} \right) = 35 + 0.87 = 35.87$$

2) Overweight and obesity may increase the risk of many health problems, including diabetes, heart disease, and certain cancers. The basic reason behind is the laziness, eating more junk foods and less physical exercise. The school management give instruction to the school to collect the weight data of each student.

During medical check of 35 students from Class X- A, there weight was recorded as follows:

Weight (in kg)	**No. of Students**
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

i) Find the median class of the given data.

ii) Find the modal class of the given data.

iii) Calculate the median weight of the given data.

OR

iv) Find the mean of the given data.

Ans: i)

Weight (in kg)	No. of Students	cf
Below 38	0	0
38 – 40	3	3
40 – 42	2	5
42 – 44	4	9
44 – 46	5	14
46 – 48	14	28
48 – 50	4	32
50 – 52	3	35

Here, n = 35 So, $\frac{n}{2}$ = 17.5

The cumulative frequency, just greater than 17.5, is 28 which belongs to class 46 – 48.

So, the median class is 46 – 48.

ii) The highest frequency in the given data is 14, which belongs to class 46-48.

So, modal classis 46 – 48.

iii) Here, l = 46, cf = 14, f = 14, h = 6

Now, $Median = l + \left(\frac{\frac{n}{2} - cf}{f} \times h \right) \Rightarrow Median = 46 + \left(\frac{17.5 - 14}{14} \times 2 \right)$

$\Rightarrow Median = 46 + \left(\frac{3.5}{14} \times 2 \right) = 46 + 0.5 = 46.5$

OR

iv)

Weight (in kg)	Class mark 'x'	'f'	fx
38 – 40	39	3	117
40 – 42	41	2	82
42 – 44	43	4	172
44 – 46	45	5	225
46 – 48	47	14	658
48 – 50	49	4	196
50 – 52	51	3	153
Total		**35**	**1603**

$Mean, \bar{x} = \frac{\sum fx}{\sum f} = \frac{1603}{35} = 45.8$

3) A group of students decided to make a project on Statistics. They are collecting the heights (in cm) of their 51 girls of Class X-A, B and C of their school. After collecting the data, they arranged the data in the following less than cumulative frequency distribution table form:

Height (in cm)	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

i) What is the lower limit of median class?

$$n = 51. \text{ So, } \frac{n}{2} = \frac{51}{2} = 25.5 .$$

This observation lies in the class 145 - 150.

Answer: 145

ii) What is the upper limit of modal class?

Answer: 150

iii) What is the mean of lower limits of median and modal class?

Answer: (i) 145

OR

iv)What is the median?

l (the lower limit) = 145 cf = 11,

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f} \right) \times h$$

$$\text{Median} = 145 + \left(\frac{25.5 - 11}{18} \right) \times 5$$

Answer: 149.03 cm

4) A group of students went to another city to collect the data of monthly consumptions (in units) to complete their Statistics project. They prepare the following frequency distribution table from the collected data gives the monthly consumers of a locality.

Monthly consumption (in units)	No. of consumers
65 - 85	4
85 -105	5
105 - 125	13
125 - 145	20
145 - 165	14
165 - 185	8
185 - 205	4

i) What is the lower limit of median class?

$$\text{Here, } \Sigma f_i = n = 68 \text{ then } \frac{n}{2} = \frac{68}{2} = 34$$

Answer: 125

ii) What is the lower limit of modal class?
Which lies in interval 125 - 145
Answer: 125
iii) Find the median:
Answer: 145

(OR)

iv) Find the mean:
Answer: 20

5)

Students per teacher	Number of School	Cf
20 - 25	**5**	**5**
25 – 30	**15**	**20**
30 - 35	**25**	**45**
35 - 40	**30**	**75**
40 - 45	**15**	**90**
45 - 50	**10**	**100**

Sol :

i) We prepare following cumulative frequency table to find median class.

$N = 100, \quad N/2 = 50$

Cumulative frequency just greater than N/2 = 50 is 75 and the corresponding class is 35-40.

Thus median class is 35 - 40 and upper limit is 40.

ii) Class 35-40 has the maximum frequency 30, So, this is model class.

Lower limit of this class = 35

iii) Median = l + [(n/2 – cf)/ f] x h

= 35 + [(50 – 45) / 30] x 5

= 35 + 5/6 = 35. 83

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CHAPTER 14-PROBABILITY

MULTIPLE CHOICE QUESTIONS

1. A bag has 5 white marbles, 8 red marbles and 4 purple marbles. If we take a marble randomly, then what is the probability of not getting purple marble? (AO1)
   (a) 0.5 (b) 0.66 (c) 0.08 (d) 0.77

2. Two dice are thrown simultaneously. What is the probability of getting doublet? (AO2)
   (a) 1/36 (b) 1/6 (c) 5/6 (d) 11/36

3. A box contains cards numbered 9 to 53. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square is (AO2)
   (a) 1/45 (b) 2/15 (c) 4/45 (d) 1/9

2. A card is selected from a deck of 52 cards. The probability of being a red face card is (AO1)
   (a) 3/26 (b) 3/13 (c) 2/13 (d) 1/2

3. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is (A02)
   (a) 7 (b) 14 (c) 21 (d) 28

4. Two dice are thrown at the same time and the product of numbers appearing on them is noted. The probability that the product is a prime number is (AO2)
   (a) 1/3 (b) 1/6 (c) 1/5 (d) 5/6

5. A ticket is drawn at random from a bag containing tickets numbered from 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is (AO2)
   (a) 1/5 (b) 3/5 (c) 4/5 (d) 1

6. Two different dice are thrown together. The probability of getting the sum of the two numbers less than 7 is: (AO1)
   (a) 5/12 (b) 7/12 (c) 12/5 (d) 3/11

ASSERTION AND REASON QUESTIONS

In the following questions 1 and 4, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

1. Assertion (A): The probability of getting exactly one head in tossing a pair of coins is 1/2.
   Reason (R): The sample space of two coin tossed is = {HH, TT, HT, TH} = 4

2. Assertion (A): The probability of winning a game is 0.4, then the probability of losing it, is 0.6.
   Reason (R): P(E) + P (not E) = 1

3. Assertion (A) : When two coins are tossed simultaneously then the probability of getting no tail is 1/4.

Reason (R): The probability of getting a head (i.e., no tail) in one toss of a coin is 1/2.

4. Assertion (A): In a simultaneous throw of a pair of dice. The probability of getting a double is 1/6.

Reason (R): Probability of an event may be negative.

2 MARKS QUESTIONS

1. Find the probability of getting 53 Fridays in a leap year. (AO1)
2. One card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn is (i) either a red card or a king, (ii) neither a red card nor a queen.(AO1)
3. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of a red ball, find the number of blue balls in the bag. (AO2)

4. Cards numbered 1 to 30 are put in a bag. A card is drawn at random from this bag. Find the probability
that the number on the drawn card is
(i) not divisible by 3.(AO1)
(ii) a prime number greater than 7.(AO1)

5. Cards, marked with numbers 5 to 50, are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken card is
(i) a prime number less than 10. (ii) a number which is a perfect square. (AO1)

3 MARK QUESTIONS

1. Two coins are tossed simultaneously. What is the probability of getting
(i) At least one head? (ii) At most one tail? (iii) A head and a tail? (AO1)

2. All the black face cards are removed from a pack of 52 playing cards. The reaming cards are well shuffled and then a card is drawn at random. Find the probability of getting (AO1)
(i) face card (ii) red card (iii) black card.

3.Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is (AO1)
(i) at least 9 (ii) 7 (iii) less than or equal to 6

4. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.(AO2)

5. Two dice are thrown simultaneously. What is the probability that (a) 6 will not come up on Either of them? (b) 6 will come up on at least one (c) 6 will come up at both dice (AO1)

5 MARKS QUESTIONS

1.From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are removed. From the remaining a card is drawn at random. Find the probability that the card drawn is

i) a black queen ii) a red card iii) a face card iv) a spade card (AO1)

2. A box contains cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears

i) a one digit number. ii) a number divisible by 5.

iii) an odd number less than 30. iv) a composite number between 50 and 70. (AO1)

3. A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a

i) triangle ii) square iii) square of blue colour iv) triangle of red colour (AO1)

CASE STUDY BASED QUESTIONS

1. Cards marked with the numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from this box. Find the probability that the number on the card is

i) an even number (AO1)

ii) a number less than 14 (AO1)

iii) a number which is a perfect square (AO2)

iv) a prime number less than 20. (AO1)

2. Tushara took a pack of 52 cards. She kept aside all the black face cards and shuffled the remaining cards well.

Based on the above information answer the following questions.

i) Write the number of total possible outcomes. (AO1)

ii) She draws a card from the well-shuffled pack of remaining cards. What is the probability that the card is a face card? (AO1)

iii) Write the probability of drawing a black card. (AO1)

OR

iv) What is the probability of getting neither a black card nor an ace card? (AO1)

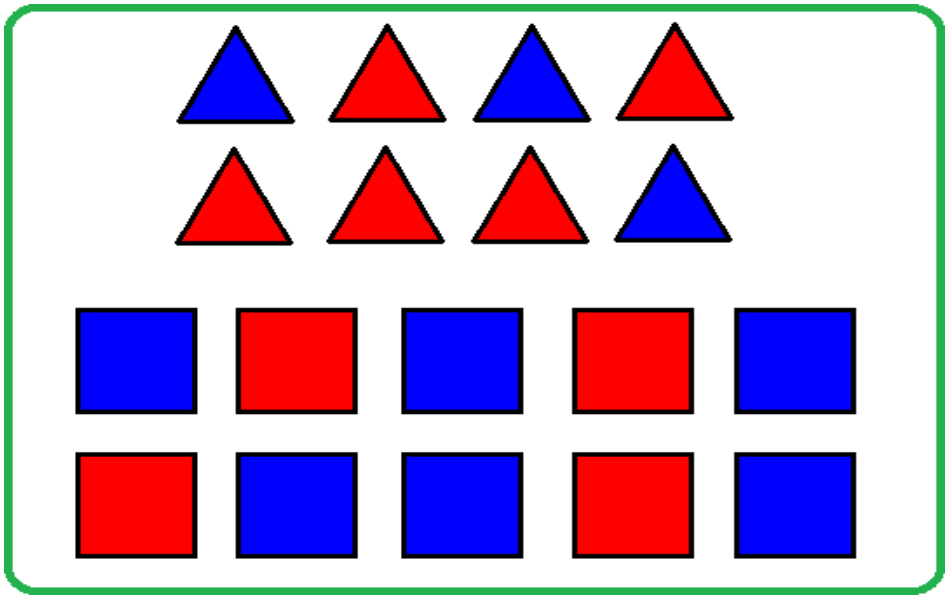
3) A, B, C, D and E are five friends. They prepared some numbered cards with labelled from 11 to 60 and then they put all the number cards in the empty box. In this game, every friend was asked to pick the card randomly and after each draw, card was replaced back in the box.

i) Find the probability that the number on the drawn card is an odd number (AO1)
ii) Find the probability that the number on the drawn card is a perfect square number (AO2)
iii) Find the probability that the number on the drawn card is:
a) divisible by 5 (AO2)
b) a prime number less than 20 (AO1)

4) Ruby and Rita are best friends. They are staying in the same colony. Both are studying in the same class and in the same school. During Winter vacation Ruby visited Rita's house to play Ludo. They decided to play Ludo with 2 dice.

i) To win a game, Ruby wanted a total of 7. What is the probability of winning a game by Ruby?(AO1)
ii) To win a game,Rita wanted 8 as the sum. What is the probability of winning a game by Rita? (AO1)
iii) What is the probability:
a) that the sum of the numbers on both the dice is divisible by 4 or 6? (AO2)
b) of getting a total of at least 10 (AO1)

5) Aditya went to shop to purchase a child's game along with his friend. He selected one child's game which has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. While checking the game, one piece islost at random.



i) Find the probability that lost piece is:

a) square b) triangle (AO1)

ii) Find the Probability that lost piece is square of blue color? (AO1)

iii) Find the Probability that lost piece is triangle of red color? (AO1)

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SOLUTIONS: CHAPTER 14-PROBABILITY
MULTIPLE CHOICE QUESTIONS

1. A bag has 5 white marbles, 8 red marbles and 4 purple marbles. If we take a marble randomly, then what is the probability of not getting purple marble?
   (a) 0.5 (b) 0.66 (c) 0.08 (d) 0.77
   Ans: (d) 0.77
   Total number of purple marbles = 4
   Total number of marbles in bag = 5 + 8 + 4 = 17
   Probability of getting not purple marbles = 13/17 = 0.77

2. Two dice are thrown simultaneously. What is the probability of getting doublet?
   (a) 1/36 (b) 1/6 (c) 5/6 (d) 11/36
   Ans: (b) 1/6
   Number of Possible outcomes are 36
   Number of favourable outcomes = 6
   Probability = 6/36 = 1/6

3. A box contains cards numbered 9 to 53. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square is :
   (a) 1/45 (b) 2/15 (c) 4/45 (d) 1/9
   Ans. (d) 1/9
   P(perfect Square)= 5/45 = 1/9

4. A card is selected from a deck of 52 cards. The probability of being a red face card is
   (a) 3/26 (b) 3/13 (c) 2/13 (d) ½
   Ans: (a) 3/26
   Total number of red face cards = 6
   ∴ Probability of being a red face card = 6/52 = 3/26

5. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is
   (a) 7 (b) 14 (c) 21 (d) 28
   Ans: (b) 14
   Total number of eggs = 400
   Probability of getting a bad egg P(E) = 0.035
   Consider x as the number of bad eggs
   P(E) = Number of bad eggs/ Total number of eggs
   Substituting the values
   0.035 = x/400 ⇒ 35/1000 = x/400 ⇒ x = 35/1000 x 400
   ⇒ x = 140/10 ⇒ x = 14

6. Two dice are thrown at the same time and the product of numbers appearing on them is noted. The probability that the product is a prime number is
   (a) 1/3 (b) 1/6 (c) 1/5 (d) 5/6
   Ans: (b) 1/6
   Total number of possible outcomes = 36

Now for the product of the numbers on the dice is prime number can be have in these possible ways = (1, 2), (2, 1), (1, 3), (3, 1), (5, 1), (1, 5)
So, number of possible ways = 6
∴ Required probability = 6/36 = 1/6

7. A ticket is drawn at random from a bag containing tickets numbered from 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is
(a) 1/5 (b) 3/5 (c) 4/5 (d) 1
Ans: (a) 1/5

8.Two different dice are thrown together. The probability of getting the sum of the two numbers less than 7 is:
(a) 5/12 (b) 7/12 (c) 12/5 (d) 3/11
Ans: (a) 5/12
Total outcomes = 36
Number of outcomes in which sum of two numbers is less than 7 = 15
∴ Required probability = 15/36 = 5/12

ASSERTION AND REASON QUESTIONS

In the following questions 1 and 4, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

1) Assertion (A): The probability of getting exactly one head in tossing a pair of coins is 1/2.
Reason (R): The sample space of two coin tossed is = {HH, TT, HT, TH} = 4
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

2) Assertion (A): The probability of winning a game is 0.4, then the probability of losing it, is 0.6.
Reason (R): P(E) + P (not E) = 1
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

3) Assertion (A) : When two coins are tossed simultaneously then the probability of getting no tail is 1/4.
Reason (R): The probability of getting a head (i.e., no tail) in one toss of a coin is 1/2.
Ans: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (B).

4) Assertion (A): In a simultaneous throw of a pair of dice. The probability of getting a double is 1/6.
Reason (R): Probability of an event may be negative.
Ans: Assertion (A) is true but reason (R) is false.

2 MARK QUESTIONS

1) Find the probability of getting 53 Fridays in a leap year.

Ans: Leap year contains 366 days. $\Rightarrow$ 52 weeks + 2 days 52 weeks contain 52 Fridays.

We will get 53 Fridays if one of the remaining two days is a Friday. Total possibilities for two days are:

(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)

There are 7 possibilities and out of these there are 2 favourable cases.

$\therefore$ P (53 Fridays) = $\frac{2}{7}$

2) One card is drawn at random from a well-shuffled deck of 52 playing cards. Find the probability that the card drawn is (i) either a red card or a king, (ii) neither a red card nor a queen.

Ans: Total number of cards = 52

(i) Number of either red card or a king card = 28

Required Probability = $\frac{28}{52} = \frac{7}{13}$

(ii) Number of cards neither a red card or a queen card = 52 – 28 = 24

Required Probability = $\frac{24}{52} = \frac{6}{13}$

3) A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of a red ball, find the number of blue balls in the bag.

Ans: Let the number of blue balls be x.

Total number of balls in the bag = $5 + x$

$\therefore$ Probability of drawing a red ball = $\frac{5}{5+x}$

and probability of drawing a blue ball = $\frac{x}{5+x}$

Given probability of drawing a blue ball = 3 × probability of drawing a red ball

$\Rightarrow \ 3 \times \frac{5}{5+x} = \frac{x}{5+x} \Rightarrow 15 = x$

$\Rightarrow$ Number of blue balls = 15

4) Cards numbered 1 to 30 are put in a bag. A card is drawn at random from this bag. Find the probability that the number on the drawn card is

(i) not divisible by 3.

(ii) a prime number greater than 7.

Ans : Total possible outcomes of drawing a card from a bag out of 30 cards = 30.

(i) Favourable outcomes for a card numbered not divisible by 3 = 20 (i.e. 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, 25, 26, 28 and 29).

Probability of drawing a card numbered not divisible by 3 = 20/30 = 2/3

(ii) Favourable outcomes for a prime numbered card greater than 7 = 6 (i.e. 11, 13, 17, 19, 23 and 29)

Probability of drawing a prime number card, greater than 7 = 6/30 = 1/5

5) Cards, marked with numbers 5 to 50, are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken card is

i) a prime number less than 10. ii) a number which is a perfect square.

Ans: Total no. of cards = 46

Total no. of ways to select a card = 46

(i) Prime no. less than 10 in these cards are 5, 7

∴ No. of ways to select a prime no. less than 10 = 2.

∴ Probability that the number on the card is prime = 2/46 = 1/23

(ii) No. which is a perfect square, i.e. 9, 16, 25, 36, 49.

No. of ways to select a card with perfect square = 5.

∴ Probability = 5/46

3 MARK QUESTIONS

1) Two coins are tossed simultaneously. What is the probability of getting

(i) At least one head? (ii) At most one tail? (iii) A head and a tail?

Ans: Total number of outcomes = 4

(i) Number of outcomes with at least one head = 3

∴ Required probability = 3/4

(ii) Number of outcomes with at most one tail = 3

∴ Required probability = 3/4

(iii) Number of outcomes with a head and a tail = 2

∴ Required probability = 2/4 = 1/2

2) All the black face cards are removed from a pack of 52 playing cards. The reaming cards are well shuffled and then a card is drawn at random. Find the probability of getting

(i) face card (ii) red card (iii) black card.

Ans: When all the black face cards are removed,

Remaining number of cards = 52 – 6 = 46

(i) Number of face cards in the remaining deck = 6

∴ P(getting a face card) = 6/46 = 3/23

(ii) Number of red cards in the remaining deck = 26

∴ P (getting a red card) = 26/46 = 13/23

(iii) Number of black cards in the remaining deck = 20

∴ P (getting a black card) = 20/46 = 10/23

3) Two dice are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is

(i) at least 9? (ii) 7? (iii) less than or equal to 6?

Ans: (i) Number of outcomes with sum of the numbers is at least 9 = 10

∴ Required Probability = 10/36 = 5/18

(ii) Number of outcomes with sum of the numbers 7 = 6

∴ Required Probability = 6/36 = 1/6

(iii) Number of outcomes with sum of the numbers less than or equal to 6 = 36

∴ Required Probability = 15/36 = 5/12

4) A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is Twice that of a red ball, find the number of blue balls in the bag.

Ans: Let the number of blue balls be x. Total number of balls in the bag = 5 + x

∴ Probability of drawing a red ball = $\frac{5}{5+x}$

and probability of drawing a blue ball = $\frac{x}{5+x}$

Given probability of drawing a blue ball = 2 × probability of drawing a red ball

Number of blue balls = 10

5) Two dice are thrown simultaneously. What is the probability that

a) 6 will not come up on either of them?

b) 6 will come up on at least one?

c) 6 will come up at both dice?

Ans: Total no. of outcomes = 36

a) Number of outcomes in which 6 will not come up on either of them = 25.

∴ Required Probability = 25/36

b) Number of outcomes in which 6 will come up at least one die = 11.

∴ Required Probability = 11/36

c) Number of outcomes in which 6 will come up at both die = 1.

∴ Required Probability = 1/36

5 MARKS QUESTIONS

1) From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are removed. From the remaining a card is drawn at random. Find the probability that the card drawn is

i) a black queen ii) a red card iii) a face card iv) a spade card

Ans: From the total playing 52 cards, red coloured jacks, queen, kings and aces are removed(i.e., 2 jacks, 2 queens, 2 kings, 2 aces) ∴ Remaining cards = 52 – 8 = 44

(i) Favourable cases for a black queen are 2 (i.e., queen of club or spade)

∴ Probability of drawing a black queen = 2/44 = 1/22

(ii) Favourable cases for red cards are 26 – 8 = 18 (as 8 cards have been removed) (i.e.9 diamonds + 9 hearts)

∴ Probability of drawing a red card = 18/44 = 9/22

(iii) Favourable cases for a face card are 6 (i.e. 2 black jacks, queens and kings each)

∴ Probability of drawing a face card = 6/44 = 3/22

(iv) Favourable cases for a spade card are 13

∴ Probability of drawing a spade card = 13/44

2) A box contains cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears

i) a one digit number. ii) a number divisible by 5.

iii) an odd number less than 30. iv) a composite number between 50 and 70.

Ans: Number of cards in the box = 65

i) Cards bearing one digit numbers are 6, 7, 8, 9

Number of such cards = 4

Probability of card bears a one digit number = $\frac{4}{65}$

ii) B = Number on the cards is divisible by 5

∴ Cards favourable to B are 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70

$\therefore$ P(B) = $\frac{13}{65} = \frac{1}{5}$

iii) C = Cards with an odd number less than 30 i.e. 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29

P(C) = $\frac{12}{65}$

iv) D : Card with composite number between 50 and 70

i.e. 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68, 69

$\therefore$ P(D) = $\frac{15}{65} = \frac{3}{13}$

3) A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a

i) triangle ii) square iii) square of blue colour iv) triangle of red colour

Ans: Total number of pieces = 8 + 10 = 18

i) No. of triangles = 8. Hence, P(triangle is lost) = 8/18 = 4/9

ii) No. of squares = 10. Hence, P(square is lost) = 10/18 = 5/9

iii)No. of squares of blue colour = 6. So, P(square of blue colour is lost) = 6/18 = 1/3

iv) No. of triangles of red colour = 8 – 3 = 5. So, P(triangle of red colour is lost) = 5/1

CASE STUDY BASED QUESTIONS

1) Ans: Total number of cards = 100

(i) Number of cards bearing even number = 50

$\therefore$ Probability of drawing an even number = 50/100 = 1/2

(ii) Total cards with number less than 14 = 12

$\therefore$ Probability of drawing a number less than 14 = 12/100 = 3/25

(iii) Total perfect squares {4, 9, 16, 25, 36, 49, 64, 81, 100} = 9

$\therefore$ Probability of drawing a perfect square = 9/100

(iv) Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19

$\therefore$ Probability of drawing a prime number less than 20 is = 8/100 = 2/25

2)**Ans**: (i) Total possible outcomes = 52 – 6 = 46

ii) Number of favourable outcomes = 6

P(face card) = 6/46 = 3/23

iii) Number of black cards in the shuffled cards = 13 + 7 = 20

P(black card) = 20/46 = 10/23

OR

Number of black cards and ace = 20 + 2 = 22

$\therefore$ Number of favourable outcomes = 46 – 22 = 24

P(neither a black card nor an ace) = 24/46 = 12/23

3) A, B, C, D and E are five friends. They prepared some numbered cards with labelled from 11 to 60 and then they put all the number cards in the empty box. In this game, every friend was asked to pick the card randomly and after each draw, card was replaced back in the box.

i) Find the probability that the number on the drawn card is an odd number
ii) Find the probability that the number on the drawn card is a perfect square number
iii)) Find the probability that the number on the drawn card is:
a) divisible by 5 b) a prime number less than 20

Ans: i) Total number of outcomes = 50
Total number of odd number cards = 25
Hence probability of getting an odd number card = 25/50 = 1/2
ii) Total number of perfect square cards = 4
16, 25, 36 and 49.
Hence probability of getting a heart card = 4/50 = 2/25
iii) a) Total number of cards divisible by 5 = 10
Hence probability of cards divisible by 5 = 10/50 = 1/5
b) Total number of prime number less than 20 = 4
Hence probability of getting prime number less than 20 = 4/50 = 2/25

4. Ruby and Rita are best friends. They are staying in the same colony. Both are studying in the same class and in the same school. During winter vacation Ruby visited Rita's house to play Ludo. They decided to play Ludo with 2 dice.

i) To win a game, Ruby wanted a total of 7. What is the probability of winning a game by Ruby?
ii)To win a game, Rita wanted 8 as the sum. What is the probability of winning a game by Rita?

iii) What is the probability:

a) that the sum of the numbers on both the dice is divisible by 4 or 6?

b) of getting a total of at least 10

Ans: i) 1/6

ii) 5/36

iii) a) 7/18

b) 1/6

5.

i) Find the probability that lost piece is:

a) square b) triangle

ii) Find the Probability that lost piece is square of blue color.

iii) Find the Probability that lost piece is triangle of red color.

Ans: i) a) 5/9

b) 4/9

ii) 1/3

iii) 5/18

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