



CHAPTER 3 : FINDING COMMON GROUND



SUMMARY OF CONCEPTS

1. HIGHEST COMMON FACTOR (HCF / GCD)

- The HCF (or GCD) of two or more numbers is the greatest of their common factors.

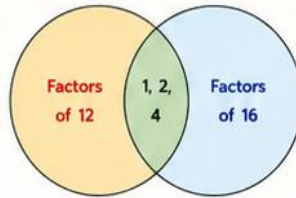
Example: HCF of 12 and 16

Factors of 12 : 1, 2, 3, 4, 6, 12

Factors of 16 : 1, 2, 4, 8, 16

Common factors: 1, 2, 4

HCF = 4



Finding HCF using Prime Factorisation

Steps:

- Find prime factorisation of the numbers.
- Take the common prime factors.
- Take the smallest power of each common prime.

Example:

$$12 = 2^2 \times 3 \quad 16 = 2^4$$

Common prime: 2

Smallest power: 2^2

$$\text{HCF} = 2^2 = 4$$

2. LOWEST COMMON MULTIPLE (LCM)

- The LCM of two or more numbers is the least (smallest) of their common multiples.

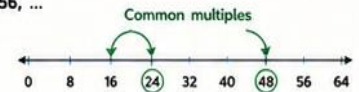
Example: LCM of 6 and 8

Multiples of 6 : 6, 12, 18, 24, 30, 36, 42, 48, ...

Multiples of 8 : 8, 16, 24, 32, 40, 48, 56, ...

Common multiples: 24, 48, 72, ...

LCM = 24



Finding LCM using Prime Factorisation

Steps:

- Find prime factorisation of the numbers.
- Take all prime factors (common and non-common).
- Take the greatest power of each prime.

Example:

$$6 = 2 \times 3 \quad 8 = 2^3$$

All primes: 2, 3

Greatest powers: $2^3, 3^1$

$$\text{LCM} = 2^3 \times 3 = 24$$

3. RELATION BETWEEN HCF AND LCM

For any two positive integers a and b:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Example: For 12 and 18

HCF = 6, LCM = 36

$$6 \times 36 = 216 = 12 \times 18 \quad \checkmark$$

4. QUICK PROCEDURE (COMMON DIVISION METHOD)

Divide both numbers by common factors successively.

To find HCF → Multiply the common factors only.

To find LCM → Multiply the common factors and the last remaining numbers.

Example: 84 and 180

$$\begin{array}{r|rr} 2 & 84 & 180 \\ 2 & 42 & 90 \\ 3 & 21 & 45 \\ & 7 & 15 \end{array} \quad \begin{array}{l} \text{HCF} = 2 \times 2 \times 3 \\ = 12 \end{array}$$

$$\text{LCM} = 2 \times 2 \times 3 \times 7 \times 15 = 1260$$

5. SPECIAL CASES & PATTERNS

- If one number is a factor (or multiple) of the other, HCF is the smaller number.
- Consecutive numbers → HCF = 1
- Co-prime numbers → HCF = 1
- If both numbers are doubled, HCF also doubles.
- LCM of co-prime numbers = product of the numbers.
- In general:
 - Multiples of a number share that number as a common factor.
 - Common multiples are multiples of the LCM.

6. MULTIPLICATION OF INTEGERS

Signs of Numbers	Product	Rule	Properties
$+$ $\times$ $+$	$+$	Same signs → Positive	Commutative : $a \times b = b \times a$
$-$ $\times$ $-$	$+$	Same signs → Positive	Associative : $a \times (b \times c) = (a \times b) \times c$
$+$ $\times$ $-$	$-$	Different signs → Negative	Distributive : $a \times (b + c) = a \times b + a \times c$
$-$ $\times$ $+$	$-$	Different signs → Negative	Identity : $a \times 1 = a, a \times (-1) = -a$
			Zero : $a \times 0 = 0$

7. DIVISION OF INTEGERS

Signs of Numbers	Quotient	Rule
$+$ $\div$ $+$	$+$	Same signs → Positive
$-$ $\div$ $-$	$+$	Same signs → Positive
$+$ $\div$ $-$	$-$	Different signs → Negative
$-$ $\div$ $+$	$-$	Different signs → Negative

Key Points

- Division by zero is not defined.
- $a \div b = a \times \left(\frac{1}{b}\right), b \neq 0$
- $a \div 1 = a, a \div (-1) = -a$
- $0 \div a = 0 (a \neq 0)$

8. REAL-LIFE APPLICATIONS

Tiling / Packing



Largest square tile or pack size = HCF
(e.g. room 12 ft $\times$ 16 ft
→ HCF = 4 ft tiles)

Packing / Grouping



Equal weight bags with least number = HCF
(e.g. 84 kg & 108 kg
→ HCF = 12 kg)

Meeting / Events



Events repeat together after = LCM
(e.g. every 7 & 10 days
→ LCM = 70 days)

Games / Patterns



Common step/jump size or pattern repeat = LCM
(e.g. Jump Jackpot game)

9. QUICK REFERENCE

Prime Factorisation Steps

- Divide the number by the smallest prime factor.
- Divide the quotient again by a prime factor.
- Continue till the quotient is 1.
- Multiply all the prime factors.

To Find HCF & LCM (Using Prime Factors)

- HCF → Multiply common primes with their smallest powers.
- LCM → Multiply all primes with their greatest powers.



Use HCF for largest size or least number of groups.



Use LCM for repeating events or common cycles.



Follow sign rules in multiplication and division.



HCF $\times$ LCM = Product of the numbers.



Prime factorisation is the key to quick and easy solutions!

FINDING THE COMMON GROUND

WORKSHEET-1

Section A: MCQs (Part-1) ($10 \times 1 = 10$ marks)

1. The common factors of 12 and 16 are:

- a) 1, 2, 4 b) 2, 4, 8 c) 1, 3, 4 d) 1, 2, 8

2. A prime number has exactly:

- a) 1 factor b) 2 factors c) 3 factors d) 4 factors

3. Prime factorisation of 105 is:

- a) $3 \times 5 \times 7$ b) 5×21 c) 15×7 d) 35×3

4. Which of the following is NOT a factor of 225?

- a) 15 b) 45 c) 75 d) 100

5. The HCF of 45 and 75 is:

- a) 5 b) 10 c) 15 d) 25

6. Co-prime numbers have HCF:

- a) 0 b) 1 c) 2 d) Same as smaller number

7. The Lowest Common Multiple means:

- a) Greatest factor b) Smallest common multiple
c) Largest multiple d) Prime multiple

8. The LCM of 6 and 8 is:

- a) 12 b) 24 c) 48 d) 72

9. The shortest toran length possible for strips of 6 cm and 8 cm is:

- a) 12 cm b) 18 cm c) 24 cm d) 48 cm

10. If one number is a multiple of another, then the LCM is:

- a) Smaller number b) Product c) Larger number d) 1

MCQs (Part-2) ($10 \times 1 = 10$ marks)

11. The HCF of 24 and 36 is:

- a) 6 b) 12 c) 18 d) 24

12. The LCM of 8 and 12 is:

- a) 12 b) 24 c) 36 d) 48

13. If HCF of two numbers is 5 and their product is 500, then their LCM is:

- a) 50 b) 100 c) 25 d) 20

14. Which of the following numbers is a factor of both 45 and 60?

- a) 5 b) 9 c) 12 d) 15

15. If two numbers are co-prime, their HCF is:

- a) 0 b) 1 c) 2 d) Their LCM

16. Three bells ring at intervals of 9 minutes, 12 minutes, and 15 minutes. If they ring together at 8:00 a.m., after how many minutes will they ring together again?

- (a) 90 minutes (b) 180 minutes (c) 60 minutes (d) 45 minutes

17. Which of the following pairs of numbers is co-prime?

- (a) 18 and 24 (b) 21 and 28 (c) 25 and 36 (d) 27 and 45

18. A teacher has 42 red pens and 56 blue pens. She wants to make identical gift packs using all the pens without any left over. What is the greatest number of gift packs she can make?

- (a) 7 (b) 14 (c) 21 (d) 28

19. Which of the following numbers has the prime factorisation $2^3 \times 3^2 \times 5$?

- (a) 240 (b) 300 (c) 360 (d) 420

20. The HCF of two numbers is 8, and one of the numbers is 40. Which of the following cannot be the other number?

- (a) 24 (b) 56 (c) 72 (d) 30

WORKSHEET-2

Section B: Short Answer ($10 \times 2 = 20$ marks)

1. Find the HCF of 112 and 84 using prime factorisation.

2. How many 4 ft by 4 ft square tiles will Sameeksha need to cover her 12 ft by 16 ft room's floor?

3. Two treasures are buried at numbers 12 and 28 on a number line. Jumpy wants to jump starting at 0 with the longest possible jump length to land exactly on both treasures. What is the longest jump size?

4. If one number is a factor of the other, what will the HCF of the two numbers be?

5. Find the LCM of 15,20 by prime factorisation.

6. Write the prime factorisation of 300.

7. What will be the HCF of two consecutive numbers ?

8. The factors of 84 are: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84. The factors of 108 are:

1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108. What is the HCF?

9. Check whether 2×7 is a factor of 210 ?

10. Using prime factorisation, find the HCF of $3 \times 3 \times 5 \times 7 \times 7$ and $12 \times 7 \times 11$.

Section C: Short Answer ($6 \times 3 = 18$ marks)

11. If the product of two numbers is 950, their HCF is 5. Find The LCM of these two numbers .

12. Verify: $\text{HCF}(30, 72) \times \text{LCM}(30, 72) = 30 \times 72$

13. Find the prime factorisation of 1200

14. . Find the HCF of 300 and 150 using the division method.

15. Find the LCM of 24,30 and 36.

16. Meena has 24 toffees and 36 chocolates. She wants to pack them into small bags so that each bag has the same number of toffees and the same number of chocolates, with nothing left over

Section D: Long Answer ($4 \times 5 = 20$ marks)

17. A shopkeeper has two ropes, one 48 m long and the other 60 m long. He wants to cut both ropes into pieces of equal length with no rope wasted. What is the longest possible length of each piece?

18. Bus A leaves a bus stand every 18 minutes and Bus B leaves every 24 minutes. Both buses left the stand together at 6:00 a.m. At what time will they next leave together?

19. Two numbers have an HCF of 12 and an LCM of 360. One of the numbers is 72. Find the other number. Also verify your answer by checking that the product of the two numbers equals the product of their HCF and LCM.

20. Lekhana bought 84 kg and 108 kg of rice from two farms. She wants to pack it in bags of equal weight using as few bags as possible. What should the bag weight be? How many bags of each type will she need? Show full working.

SECTION E: Case Study Based Questions ($4 \times 2 = 8$ marks)

21. The craft competition

Priya's school is holding a craft fair. She has three rolls of ribbon — one red roll of **72 cm**, one blue roll of **90 cm**, and one yellow roll of **120 cm**. She wants to cut all the ribbons into pieces of equal length so that no ribbon is wasted, and each piece is as long as possible.



A) What is the greatest length of each piece of ribbon that Priya can cut? Show your working using prime factorisation.

B) How many pieces will she get from each ribbon — the red, the blue, and the yellow — if she cuts them at the length found in part (A)?

C) If Priya instead wants pieces of exactly 6 cm each, will there be any ribbon wasted? Justify your answer and state how many total pieces she will get from all three rolls together.

22. Rohan is preparing goody bags for his birthday party. He has 48 chocolates, 60 lollipops, and 36 toffees. He wants to divide them equally into as many bags as possible, with each bag having the same number of each type of sweet and nothing left over.



A) Find the maximum number of goody bags Rohan can make. What is the HCF of 48, 60, and 36?

B) How many chocolates, lollipops, and toffees will go into each bag?

C) Rohan's mother adds 24 more chocolates to the collection. Does the maximum number of bags change now? Find the new HCF of all the chocolates (72), lollipops (60), and toffees (36) and explain.

Self Assessment:

I understood all the concepts ☐ Yes ☐ No

I need more practice ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood all the concepts ☐ Yes ☐ No

He/She need more practice ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers of Worksheet-1

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	A	B	A	D	C	B	B	B	C	C
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans.	B	B	B	A	B	b	c	b	c	d

Answers of Worksheet-2

Q.No.	1	2	3	4	5	6
	28	12 tiles	4	Smaller number	60	$2^2 \times 3 \times 5^2$
	7	8	9	10	11	12
Ans.	1	12	Yes	7	190	Verified ($2160=2160$)
	13	14	15	16	17	18
	$2^4 \times 3 \times 5^2$	150	360	12 bags	12 m	72 min (7:12 a.m.)
	19	20				
	60	12 kg; 7 & 9bags	(A)6 cm (B)12,15,20 (C)47pieces	(A)12 bags (B)4,5,3 (C)HCF=12		