4 EXPLORING SOME GEOMETRIC THEMES

In This Chapter
- ✓ Fractals (self-similar shapes)
- ✓ Visualising Solids in Different Ways

We explore two geometric themes in this chapter:
1. **Fractals** – self-similar shapes that repeat the same pattern at smaller and smaller scales.
2. **Visualising Solids** – understanding the same solid from different views and cross-sections.

1. FRACTALS

Key Idea

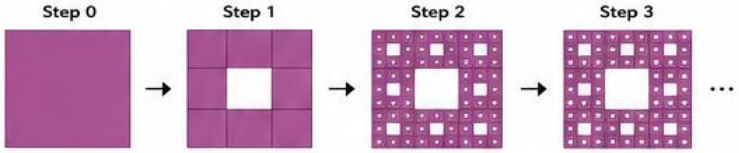
A **fractal** is a shape that shows **self-similarity** – the same or similar pattern appears again and again at smaller and smaller scales.

Examples in Nature

Ferns, trees, clouds, coastlines, mountains, lightning and many more.

A. SIERPINSKI CARPET

Start with a square. Divide it into 9 equal squares and remove the central square. Repeat the same process on the remaining 8 squares, and so on.



Number of Squares that Remain (R_n)

Each remaining square produces 8 squares in the next step.

$$R_{n+1} = 8\,R_n$$

$$R_0 = 1 \Rightarrow R_n = 8^n$$

Number of Holes (H_n)

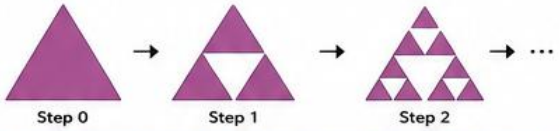
Each remaining square creates a new hole and all previous holes remain.

$$H_{n+1} = H_n + R_n$$

$$H_0 = 0,\ H_1 = 1,\ H_2 = 1+8,\ H_3 = 1+8+8^2, \ldots$$

B. SIERPINSKI TRIANGLE (GASKET)

Start with an equilateral triangle. Join midpoints of its sides to form 4 smaller congruent triangles and remove the central one. Repeat on the 3 remaining triangles.



☆ Joining the midpoints of an equilateral triangle divides it into 4 identical equilateral triangles.
(Hint: Corner triangles are isosceles.)

Figure it Out
1. Draw the first few steps (at least till Step 2).
2. Find the number of holes and triangles that remain at each step.
3. Find the area of the region remaining at the n^{th} step.
(Take the starting triangle to have area = 1 sq. unit.)

C. KOCH SNOWFLAKE

Start with an equilateral triangle. Each side is divided into 3 equal parts. On the middle part, an outward equilateral triangle is built and the middle part is removed. Repeat on every side of the new shape, and so on.

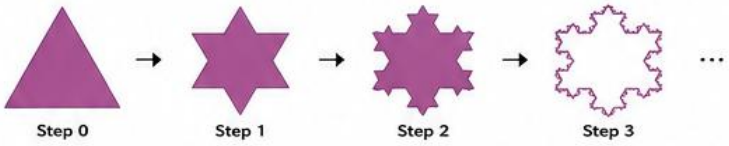


Figure it Out
1. Draw the first few steps (at least till Step 2).
2. Find the number of sides at the n^{th} step.
3. Find the perimeter at the n^{th} step.
(Take the starting triangle to have side length = 1 unit.)

FRACTALS IN ART

Fractal patterns have been used in art and architecture for centuries!

- **Indian temples:** Structures built of smaller copies of the whole form.
- **African art:** Traditional patterns on textiles (e.g., Nigerian Fulani wedding blankets).
- **Islamic art:** Intricate repeating fractal-like geometric motifs.


Kandariya Mahadev Temple, Khajuraho, Madhya Pradesh (c. 1025 CE)


Nigerian Fulani Wedding Blanket

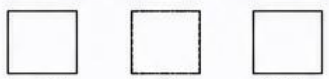

Islamic Geometric Motif

2. VISUALISING SOLIDS IN DIFFERENT WAYS

We can look at the same solid in different ways:

A. DIFFERENT VIEWS

A solid looks different from different directions.



B. CROSS-SECTIONS

A slice (cut) of a solid gives a 2D shape.

C. EXPLORE & SKETCH

- Think, imagine, cut and draw different views and cross-sections to understand solids better.

KEY DEFINITIONS

- **Fractal:** A shape that shows self-similarity at different scales.
- **Self-similarity:** Similar pattern repeats at smaller and smaller sizes.
- **Square Hole / Triangle Hole:** A part removed at each step in constructing a fractal.
- **Cross-section:** The 2D shape obtained by slicing a solid with a plane.

IMPORTANT FORMULAS

Sierpinski Carpet

$$R_{n+1} = 8\,R_n \Rightarrow R_n = 8^n$$
$$H_{n+1} = H_n + R_n \quad (\text{with } H_0 = 0)$$

Sierpinski Triangle

$$R_{n+1} = 3\,R_n \Rightarrow R_n = 3^n$$
$$H_{n+1} = H_n + R_n \quad (\text{with } H_0 = 0)$$

Koch Snowflake (perimeter)

$$P_{n+1} = \frac{4}{3}P_n \Rightarrow P_n = \left(\frac{4}{3}\right)^n P_0$$

KEY TAKEAWAYS

- ✓ Fractals are everywhere – in nature, art and mathematics.
- ✓ They are built using simple rules repeated many times.
- ✓ Counting patterns (squares, triangles, holes, sides) helps us understand fractals.
- ✓ Visualising solids from different views and cross-sections builds spatial understanding.

EXPLORING SOME GEOMETRICAL THEMES

WORKSHEET-1

Section A: MCQs (Part-1) (10 × 1 = 10 marks)

1. A pattern that repeats itself at different scales is called:

a) Polygon b) Fractal c) Prism d) Pyramid

2. The Koch Snowflake begins with:

a) Square b) Rectangle c) Equilateral Triangle d) Pentagon

3. The Sierpinski Gasket is formed using:

a) Square b) Circle c) Triangle d) Rectangle

4. Which property is common to fractals?

a) Reflection b) Translation c) Self-similarity d) Rotation

5. Isometric paper is made up of:

a) Squares b) Rectangles c) Equilateral triangles d) Circles

6. A net is:

a) A graph b) A flat pattern that folds into a solid c) A tessellation d) A fractal

7. A square pyramid has:

a) 3 triangular faces b) 4 triangular faces c) 5 triangular faces d) 6 triangular faces

8. Which of the following is NOT a fractal?

a) Fern leaf b) Koch Snowflake c) Sierpinski Gasket d) Rectangle

9. The perimeter of a Koch Snowflake:

a) Decreases b) Remains constant c) Increases d) Becomes zero

10. Isometric drawings are mainly used to represent:

a) 2D figures b) 3D objects c) Graphs d) Maps

MCQs (Part-2) (10 × 1 = 10 marks)

11.A standard solid cube is made up of how many faces, vertices, and edges respectively?

a) 6 faces, 8 vertices, 12 edges b) 6 faces, 12 vertices, 8 edges

c) 8 faces, 6 vertices, 12 edges d) 4 faces, 4 vertices, 6 edges

12.What flat geometric shape do you get when you fully open and flatten out the curved side wall of a hollow cylinder?

a) Circle b) Triangle c) Rectangle d) Semi-circle

13.What 3D solid is constructed by rolling up a sector of a circle (like a slice of pizza) and attaching a matching circular base?

a) Cylinder b) Cone c) Sphere d) Prism

14.Which of these flat shapes has exactly three straight sides and three corners?

a) Square b) Triangle c) Circle d) Rectangle

15. How many flat faces does a standard playing dice (cube) have?

a) 4 b) 6 c) 8 d) 12

16. What shape do you get if you trace around the bottom base of a solid ice cream cone onto a piece of paper?

a) Circle b) Square c) Triangle d) Oval

17.If you look at a building from directly above, as if you are flying in an airplane, this overhead view is called the:

a) Front view b) Side view c) Top view d) Bottom view

18. What solid shape do you get if you roll up a flat rectangular sheet of paper so that its opposite ends meet?

a) Cube b) Sphere c) Cylinder d) Pyramid

19.Which of these flat shapes has exactly three straight sides and three corners?

a) Square b) Triangle c) Circle d) Rectangle

20.How many straight sides does a regular flat pentagon have?

a) 4 b) 5 c) 6 d) 8

WORKSHEET-2

Section B: Short Answer (10 × 2 = 20 marks)

1. Define a fractal.

2. What is self-similarity?

3. Name any two fractals found in nature.

4. What is an isometric drawing?

5. Define a net of a solid.

6. Name the shape used to start the construction of a Koch Snowflake.

7. Name the shape used to start the construction of a Sierpinski Gasket.

8. Why is isometric paper useful?

9. State one similarity between Koch Snowflake and Sierpinski Gasket.

10.State one difference between Koch Snowflake and Sierpinski Gasket.

Section C: Short Answer (6 × 3 = 18 marks)

11. Explain self-similarity using the example of a fern leaf.

12. Describe the first stage in constructing a Koch Snowflake.

13. Describe the first stage in constructing a Sierpinski Gasket.

14. Draw a cube using isometric dots.

15. Explain how a net helps in understanding a three-dimensional solid.

16. Draw the net of a square pyramid and label its faces.

Section D: Long Answer (4 × 5 = 20 marks)

17. Explain the construction of a Koch Snowflake up to Stage 2.

18. Explain the construction of a Sierpinski Gasket up to Stage 2.

19. Compare Koch Snowflake and Sierpinski Gasket.

20. Draw and explain the net of a square pyramid. State faces, edges and vertices.

SECTION E: Case Study Based Questions (4 × 2 = 8 marks)

21. Which solid is formed by folding the net of a square pyramid? How many triangular faces does it have?

22. How many vertices does the solid have? If each side of the square base is 6 cm, find the perimeter.

Self Assessment:

I understood all the concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood all the concepts: ☐ Yes ☐ No

He/She need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers of Worksheet-1

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	b	c	c	c	c	b	b	d	c	b
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans.	b	a	d	b	d	c	d	c	b	d

5 TALES BY DOTS AND LINES

This chapter helps us understand how the mean behaves with data.
We use dot plots to visualise and explore important ideas.

In this chapter

- Mean as the 'centre'
- Effect of including / removing values
- Unchanging Mean
- Relatively Unchanged Mean

Scan to learn more!

1. THE BALANCING ACT

Key Idea

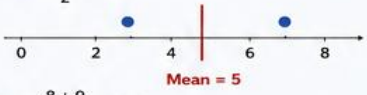
The mean (average) is the 'centre' of a collection of data. It balances the data so that the total distance of values on the left equals the total distance of values on the right.

For 2 Numbers

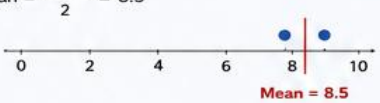
Mean is exactly halfway between the two numbers.

Example:

(3, 7) → Mean = $\frac{3+7}{2} = 5$

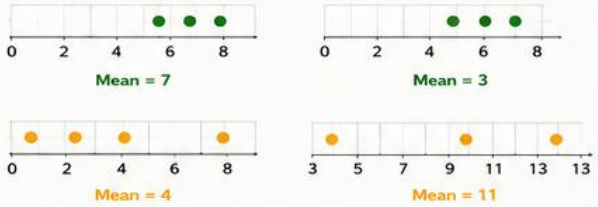


(8, 9) → Mean = $\frac{8+9}{2} = 8.5$



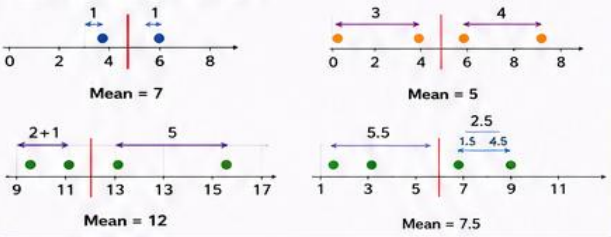
For 3 Numbers

Calculate and mark the mean.



Mean as Balance (Examples)

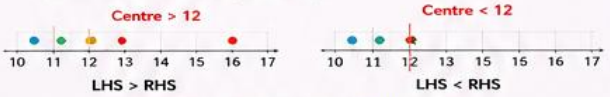
Sum of distances on LHS = Sum on RHS



Unique Centre

If we move the centre away from the mean, the two sides are no longer balanced.

Collection: 10, 10, 11, 17 **(Mean = 12)**

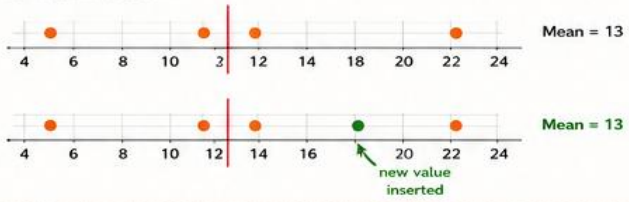


Hence, the mean is the only centre.

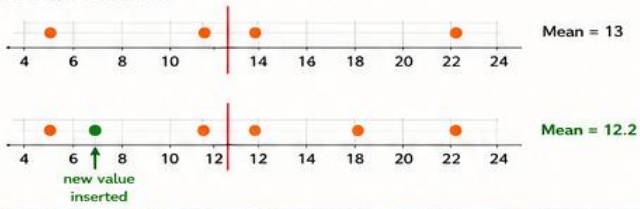
2. EFFECT OF INCLUDING / REMOVING VALUES

Including a New Value

- If a new value greater than the mean is added, the mean increases.

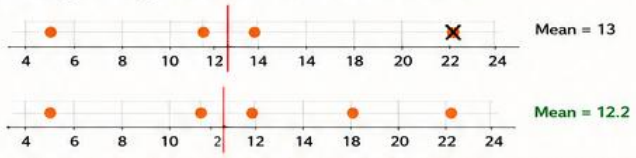


- If a new value smaller than the mean is added, the mean decreases.

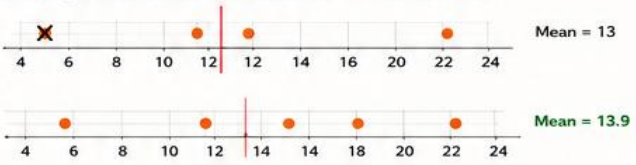


Removing a Value

- **Removing a value greater than the mean → mean decreases.**

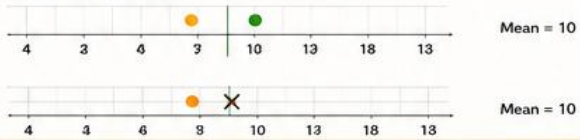


- **Removing a value smaller than the mean → mean increases.**



Adding / Removing a Value Equal to the Mean

- **Mean does not change.**



Fair-Share Interpretation

Math Talk

- Think of sharing equally (fairly) among all values.
- If one value is larger than the mean, others must share more to keep it fair.
- If one value is smaller, others share less.

3. UNCHANGING MEAN

- It is possible to include or remove values without changing the mean.
- Some possibilities:

Add one value = mean (10 added to data with mean 10)	Remove one value = mean (any one 10 removed)	Add two below & one above (e.g., 8, 8 and 12)

Similarly: add two above & one below.
There are many such combinations!

4. RELATIVELY UNCHANGED MEAN

If every value in the collection is:

1. Increased (or decreased) by a fixed number k
   → Mean also increases (or decreases) by k.
   Example: Add 10 to every value → Mean increases by 10.
   Subtract 1 from every value → Mean decreases by 1.
2. Multiplied by a fixed number k
   → Mean is also multiplied by k.
   Example: Double every value → Mean doubles.

Algebraic Rule

- Let values be $x_1, x_2, \ldots, x_n$ with mean $a = \frac{x_1 + x_2 + \cdots + x_n}{n}$
- Add k to every value: New mean $= a + k$
- Subtract k from every value: New mean $= a - k$
- Multiply every value by k: New mean $= ka$

KEY DEFINITIONS

- **Mean (Average):** Sum of all values divided by the number of values.
- **Centre (Balance Point):** A value where the sum of distances of data on the left equals the sum on the right.
- **Dot Plot:** A way to display data using dots on a number line.

IMPORTANT FORMULAS

- Mean: $\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$
- When each value changes:
  Add k → Mean $= \bar{x} + k$
  Subtract k → Mean $= \bar{x} - k$
  Multiply by k → Mean $= k\bar{x}$

KEY TAKEAWAYS

- ✓ Mean is the centre that **balances data**.
- ✓ Adding values greater than the mean increases it; smaller values decrease it.
- ✓ Removing works in the opposite way.
- ✓ Adding/removing a value equal to the mean **does not** change it.
- ✓ Mean changes predictably when all values change together.