6 ALGEBRA PLAY

We use algebra to understand number tricks, create new tricks, and explore patterns in number pyramids.

In this chapter

- Algebra tricks & puzzles
- Thinking/about 'Think of a Number' tricks
- Number pyramids

Scan to learn more!

1. THE BALANCING ACT

Key Idea

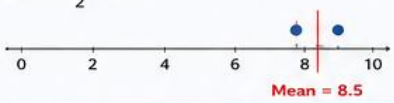
The mean (average) is the 'centre' of the data. It balances the total distance of values on the left and right.

For 2 Numbers

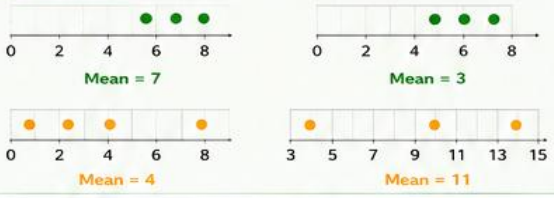
Mean is exactly halfway between the two numbers.

$(3, 7) \rightarrow \text{Mean} = \frac{3+7}{2} = 5$

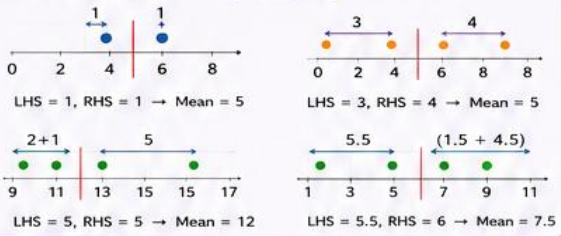
$(8, 9) \rightarrow \text{Mean} = \frac{8+9}{2} = 8.5$



For 3 Numbers – Mark the mean.

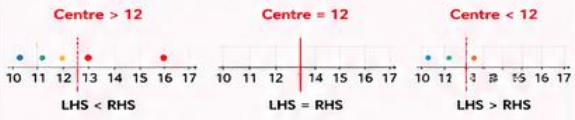


How Mean is the Centre (Examples)



Unique Centre

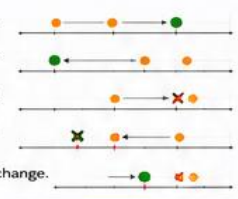
For the data 10, 10, 11, 17 (mean = 12)



Hence, the mean is the only centre.

4. HOW MEAN CHANGES

Add a value > mean	Mean increases.
Add a value < mean	Mean decreases.
Remove a value > mean	Mean decreases.
Remove a value < mean	Mean increases.
Add or remove value = mean	Mean does not change.



Relatively Unchanged Mean

If every value changes by the same amount:

- Add k to every value → Mean increases by k.
- Subtract k from every value → Mean decreases by k.
- Multiply every value by k → Mean becomes k times.
- Divide every value by k → Mean becomes $\frac{1}{k}$ times.

Math Talk

2. THINKING ABOUT 'THINK OF A NUMBER' TRICKS

(A) Always 2 Trick

	Algebraic Explanation
1. Think of a number.	1. x
2. Double it.	2. $2x$
3. Add four.	3. $2x + 4$
4. Divide by two.	4. $x + 2$
5. Subtract the original number.	5. $x + 2 - x = 2$

No matter what the starting number is, the result is always 2.

(B) Date Trick

	Algebraic Explanation
1. Think of a date (DD/MM).	Let month = M, day = D
2. Multiply the month by 5.	1. $5M$
3. Add 6.	2. $5M + 6$
4. Multiply by 4.	3. $20M + 24$
5. Add 9.	4. $20M + 33$
6. Multiply by 5.	5. $100M + 165$
7. Add the day.	6. $100M + 165 + D$

Final answer = $100M + 165 + D$

- Subtract 165 from final answer → $100M + D$
- Last two digits → D (day)
- Remaining digits → M (month)
- Example: 291 – 165 = 126 → $M = 1$, $D = 26$ → 26th January

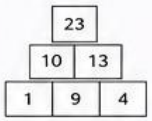
Explore & Create

- Try other dates and find the pattern.
- Change the steps (add, subtract, multiply etc.) to make new tricks.
- You can also aim for a fixed final answer like 3, 5, 10 and so on.

3. NUMBER PYRAMIDS

Rule: Each number is the sum of the two numbers directly below it.

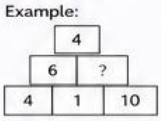
Example



How to Fill?

- To find a missing number, subtract the known number from the number above it.

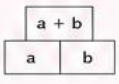
Example:



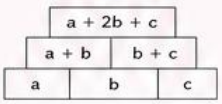
6 – 4 = 2
4 – 1 = 3
→ Missing number = 3

General Pattern

2-row pyramid



3-row pyramid



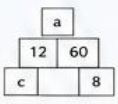
Observation

The top number = a weighted sum of the bottom row. (Middle numbers act as coefficients 1, 2, 1.)

Figure it Out

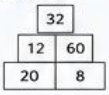
1. Fill the pyramids.

2. Given pyramid:



Equations:
$a + 12 = 60$
$12 + c = a$
$c + 8 = 60$
$\Rightarrow c = 20, a = 32$

Pyramid:



3. Without building, find top number for given bottom row.

4 13 8 7 11 → Top = 4 + 3×13 + 3×8 + 3×7 + 11 = 84

8 19 21 13 7 18 → Top = 8 + 2×19 + 2×21 + 2×13 + 2×7 + 18 = 146

(Coefficients: 1, 2, 2, ..., 2, 1)

KEY DEFINITIONS

- **Mean (Average):** Sum of all values divided by the number of values.
- **Centre (Balance Point):** Value where the sum of distances on the left equals the sum on the right.
- **Dot Plot:** Data shown as dots on a number line.
- **Number Pyramid:** Each number is the sum of the two numbers below it.

IMPORTANT FORMULAS

- Mean: $\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$
- After adding k: New mean = $\bar{x} + k$
- After subtracting k: New mean = $\bar{x} - k$
- After multiplying by k: New mean = $k\bar{x}$
- After dividing by k: New mean = $\frac{\bar{x}}{k}$

KEY TAKEAWAYS

- ✓ The mean balances data.
- ✓ Including/removing values changes the mean predictably.
- ✓ Algebra helps us understand tricks and invent new ones.
- ✓ In number pyramids, the top number is a weighted sum of the bottom row.
- ✓ Patterns in numbers can be explored and explained systematically.

ALGEBRA PLAY

WORKSHEET-1

Section A: MCQs (Part-1) (10 × 1 = 10 marks)

1. A student thinks of a number x, triples it, adds 12 and divides the result by 3. Which expression is obtained before subtracting the original number?

A. x + 12 B. x + 4 C. 3x + 4 D. x + 3

2. A number pyramid has bottom numbers 6, 9 and 4. What is the topmost number?

A. 22 B. 28 C. 25 D. 30

3. If the top row of a three-level number pyramid is represented by a + 2b + c, the coefficient of b is:

A. 1 B. 2 C. 3 D. 4

4. A 2 × 2 calendar block has top-left number n. The sum of the four numbers is:

A. 4n + 8 B. 4n + 16 C. 2n + 16 D. 4n + 14

5. To obtain the greatest product using three different digits, the largest digit should generally be used as the:

A. Tens digit B. Ones digit C. Multiplier D. Smallest digit

6. The difference between a two-digit number and its reverse is always divisible by:

A. 7 B. 9 C. 10 D. 12

7. Which branch of mathematics helps explain why number tricks always work?

A. Geometry B. Algebra C. Data Handling D. Mensuration

8.In a number pyramid, every box depends directly on:

A. The top number B. The two boxes below it
C. The left box only D. The right box only

9. A mathematical trick gives the same answer for every starting number because the:

A. Numbers are equal B. Variable gets eliminated
C. Number becomes zero D. Operation changes

10. Which of the following best develops algebraic thinking?

A. Copying formulas B. Explaining why a trick works
C. Learning tables D. Drawing figures

MCQs (Part-2) (10 × 1 = 10 marks)

11. A student thinks of a number, triples it, adds 9, divides the result by 3, and subtracts the original number. The final answer will always be

A. 1 B. 2 C. 3 D. 9

12. In a number pyramid, every box is the sum of the two boxes directly below it. If the bottom row is 6, 4, 9, then the top number is

A. 19 B. 23 C. 24 D. 27

13. A calendar has a 2 × 2 block with top-left number 18. What is the sum of the four numbers?

A. 84 B. 88 C. 92 D. 96

14. Which expression represents the top number of a three-row number pyramid whose bottom row is a, b, c?

A. $a + b + c$ B. $2a + b + c$ C. $a + 2b + c$ D. $a + b + 2c$

15. Using the digits 4, 6, and 8 exactly once to form □ □ × □, the largest product is

A. 86 × 4 B. 84 × 6 C. 64 × 8 D. 68 × 4

16. If a two-digit number is reversed and added to the original number, the result is always divisible by

A. 3 B. 5 C. 9 D. 11

17. A number trick gives the expression 5x + 20 after some operations. If the final step is to divide by 5, the resulting expression is

A. $x + 5$ B. $x + 4$ C. $5x + 4$ D. $x + 20$

18. In a number pyramid, the bottom row is 5, x, 8, and the top number is 30. The value of x is

A. 4 B. 5 C. 6 D. 7

19. If 42 is reversed to 24, then the difference between the two numbers is

A. divisible by 3 only B. divisible by 5 only
C. divisible by 9 D. not divisible by 9

20. Which of the following best describes the purpose of algebra in mathematical puzzles?

A. To avoid calculations B. To replace numbers with shapes only

C. To explain why mathematical tricks always work D. To memorize formulas

WORKSHEET-2

Section B: Short Answer (10 × 2 = 20 marks)

1. A student creates this trick:

   Think of a number → Multiply by 4 → Add 20 → Divide by 4 → Subtract the original number.

   Predict the final answer without choosing any number. Justify your answer algebraically.

2. The bottom row of a number pyramid is **7, x, 9** and the top number is **34**. Find the value of **x**.

3. A number is multiplied by 4 and then 7 is added. The result is 39. Find the number.

4. A **2 × 2** calendar block has a total sum of **100**. Find the four numbers in the block using algebra.

5. Form the greatest possible product using the digits **2, 6 and 8** (use each digit only once). Explain your choice in one or two steps.

6. A two-digit number is represented by **10a + b**. Write the algebraic expression for the number obtained after reversing its digits.

7. In an algebra grid, ★ + ▲ = 24 and ▲ = 9. Find the value of ★. Then verify your answer.

8. The perimeter of a rectangle is 74 cm. Its length is 9 cm more than its breadth. Find its dimensions.

9. The difference between two consecutive odd numbers is always 2. If their sum is 84, find the numbers.

10. A student claims that arranging the remaining digits in decreasing order always helps in obtaining the greatest product when the largest digit is used as the multiplier. Verify the claim using the digits **4, 5 and 9**.

Section C: Short Answer (5 × 3 = 15 marks)

11. A student designs this trick:

    Think of a number → Multiply by 5 → Add 15 → Divide by 5 → Subtract the original number. Prove algebraically that the final answer is always the same.

12. The bottom row of a number pyramid is **x, 8, 5**. If the top number is **35**, find **x** and complete the pyramid.

13. The cost of 5 pens and 3 pencils is ₹81. If each pencil costs ₹7, find the cost of one pen.

14. Show algebraically that the difference between any two-digit number and its reverse is always divisible by **9**.

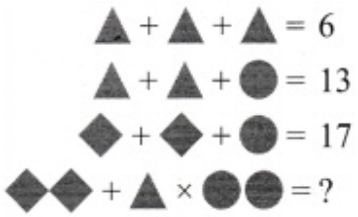
15. Using the digits **4, 6 and 8**, verify algebraically why the largest digit should be used as the multiplier to obtain the greatest product.

Section D: Long Answer (4 × 5 = 20 marks)

16. A fruit seller spends ₹1200 as a fixed daily expense and ₹18 on each fruit basket. If he sells each basket for ₹30, how many baskets must he sell to earn a profit of ₹600? Form an algebraic equation and solve it.

17. A father is 4 times as old as his son. After 8 years, the father will be twice the son's age. Find their present ages using algebra.

18. Decode the puzzle and find the final value



SECTION E: Case Study Based Questions (4 × 2 = 8 marks

19. Aarav created a number pyramid in which each box is the sum of the two boxes below it. The bottom row is **8, x, 6** and the top number is **38**. **Answer the following:** (a) Form an algebraic equation for the top number. (b) Find the value of **x**. (c) Complete the pyramid. (d) State one rule of a number pyramid.

20. Mehak chooses a **2 × 2** block from a calendar. She tells her friend that the sum of the four numbers is **124**. **Answer the following:** (a) Let the top-left number be **x**. Write the algebraic expression for the sum. (b) Find the value of **x**. (c) Write the four numbers in the block. (d) Explain why this method works for every calendar.

Self Assessment:

I understood all the concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood all the concepts: ☐ Yes ☐ No

He/She need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers of Worksheet-1

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	B	B	B	B	C	B	B	B	B	B
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	A	C	C	D	B	C	C	C

Answers of Worksheet-2

Q.No.	1	2	3	4	5	6
Ans.	5	9	8	21, 22, 28, 29	496	10b + a
Q.No.	7	8	9	10	11	12
Ans.	★ = 15	L- 23cm, B- 14cm	41, 43	486	Final answer is always **3**	x = **7** Middle row = **15, 13**; Top = **28**
Q.No.	13	14	15	16	17	18
Ans.	₹12	Difference = 9(a − b) (or 9(b − a)), hence divisible by 9	**512**	**150 baskets**	Father =32 years, Son = 8 years	44
Q.No.	19			20		
Ans.	(a) 8 + 2x + 6 = 38 (b) x = 12 (c) Middle row = 20, 18; Top = 38 (d) Each upper box is the sum of the two boxes below it.			(a) 4x + 16 = 124 (b) x = **27** (c) **27, 28, 34, 35** (d) Calendar numbers differ by **1** horizontally and **7** vertically, so the algebraic relation remains fixed.		

7 AREA

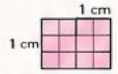
Algebra helps us explore patterns, solve puzzles and create tricks. In this chapter we use it to understand areas and number pyramids.

In This Chapter

- ✓ Area of rectangles and squares
- ✓ Why perimeter is not a measure of area
- ✓ Finding missing lengths and areas
- ✓ Area around shapes (paths)
- ✓ Number pyramids

Key Idea

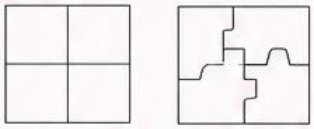
Area is the amount of surface enclosed by a shape. We measure it using unit squares.



Area of 1 square = 1 sq. cm
Area of N squares = N sq. cm or N cm²

1. RECTANGLES AND SQUARES

- Many ways to divide a square into 4 equal parts!

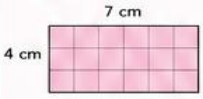


- Area can change shape but remains the same.

Area of a Rectangle

Area = length × width
(or length × breadth)
Units: sq. unit (or unit)

Example:

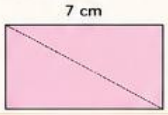


Number of unit squares = 7 × 4 = 28
Area = 28 sq. cm

Area of a Triangle

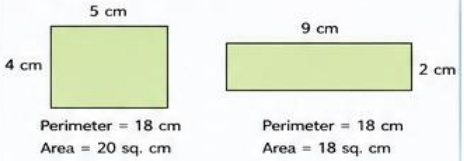
- A diagonal of a rectangle divides it into two congruent triangles.
- Area of each triangle = $\frac{1}{2}$ × length × width

Example: $\frac{1}{2} \times 7 \times 4 = 14$ sq. cm



2. WHY PERIMETER IS NOT A MEASURE OF AREA

- **Same perimeter, different areas:**



Perimeter = 18 cm
Area = 20 sq. cm

Perimeter = 18 cm
Area = 18 sq. cm

- **Larger perimeter, smaller area:**

Perimeter = 40 cm
Area = 100 sq. cm

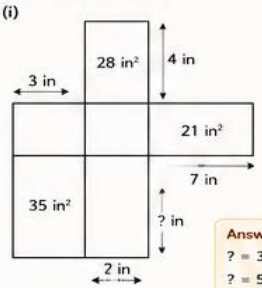
Perimeter = 50 cm
Area = 100 sq. cm

Hence, perimeter does not tell us the area.
Area depends on how much surface is enclosed inside the boundary.

3. FIGURE IT OUT

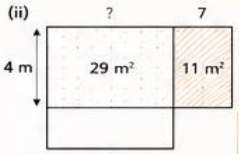
(a) Missing Sidelengths

(i)



Answers:
? = 3 in (top width)
? = 5 in (bottom width)
? = 5 in (bottom height)

(ii)

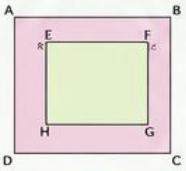


Area = 50 m²

Answers:
Top left width = 10 m
Bottom width = 17 m

4. AREAS AROUND SHAPES (PATHS)

(i) Path around a rectangular park

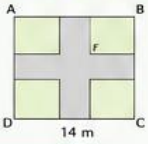


Two ways to find area of the path

Method 1:
Area(Path) = Area(ABCD) − Area(EFGH)

Method 2:
If width of path = x,
Outer: length = L, breadth = B
Inner: length = (L − 2x), breadth = (B − 2x)

(ii) Cross path in a plot

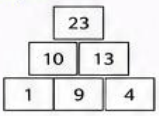


Break the path into rectangles.
If vertical width = x,
horizontal width = y,
Area(Path) = 14x + 12y − xy
(subtract the overlap xy counted twice)

5. NUMBER PYRAMIDS

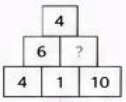
Rule: Each number is the sum of the two numbers directly below it.

Example



How to Fill?

- Work from bottom to top to find known numbers.
- Work from top to bottom to find missing numbers by subtraction.



? = 6 − 4 = 2
(top − left bottom)

General Pattern

For bottom row: a, b, c

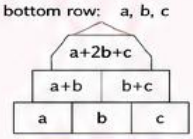


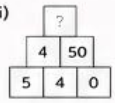
Figure it Out!

1. Fill the pyramids.

(i)

(ii)

(iii)



Write expression for top of a 4-row pyramid in terms of bottom row a, b, c, d.
Top = a + 3b + 3c + d (Coefficients: 1, 3, 3, 1)

6. HOW THE MEAN CHANGES

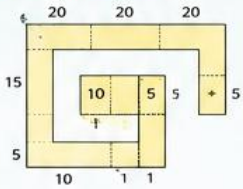
Operation on data	Effect on mean	Example (mean = 10)
Add a value > mean	Mean increases	Add 20 → mean > 10
Add a value < mean	Mean decreases	Add 2 → mean < 10
Remove a value > mean	Mean decreases	Remove 20 → mean < 10
Remove a value < mean	Mean increases	Remove 2 → mean > 10
Add or remove value = mean	Mean unchanged	Add or remove 10 → mean = 10

Relatively Unchanged Mean

- Add k to every value → mean increases by k.
- Subtract k from every value → mean decreases by k.
- Multiply every value by k → mean becomes k times.
- Divide every value by k → mean becomes $\frac{1}{k}$ times.

Math Talk

7. SPIRAL TUBE

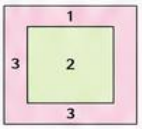


Area of tube = Sum of areas of all rectangular parts.
= 20×15 + 20×10 + 20×5 + 5×10 + 5×5 + 15×5 + 10×1 + 1×1
= 600 sq. units

Straight tube with same area
If width = 5, then required length = 600 ÷ 5 = 120 units.

8. MORE EXPLORATIONS

1. **Doubling the side of a square:**
   - Region 1 (outer square) → 4 times area
   - Region 2 (inner square) → 4 times area
   - Region 3 (frame) → 4 times area



2. **Rearranging 4 equal parts:**
   - Cut square into 4 equal parts.
   - Rearrange to get a larger square with a square hole in the centre.

KEY DEFINITIONS

- **Area:** The amount of surface enclosed by a shape, measured in square units.
- **Perimeter:** The total length of the boundary of a shape.
- **Unit Square:** A square with side 1 unit (area = 1 sq. unit).

IMPORTANT FORMULAS

- Area of rectangle = length × width
- Area of square = side × side = side²
- Area of triangle (from rectangle) = $\frac{1}{2}$ × length × width
- Path around rectangle (method 1):
  Area = Outer rectangle − Inner rectangle
- Path (cross) in a plot: Area = 14x + 12y − xy
  (x = vertical width, y = horizontal width)

KEY TAKEAWAYS

- ✓ We measure area using unit squares.
- ✓ Perimeter cannot determine area.
- ✓ Algebra helps us find unknown lengths and areas.
- ✓ In number pyramids, each number is the sum of the two below.
- ✓ Mean changes in predictable ways with operations on data.
- ✓ Patterns in numbers can be explored using algebra.