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PROPORTIONAL REASONING-2

In This Chapter

- ✓ Proportionality (quick recap)
- ✓ Ratios in maps
- ✓ Ratios with more than 2 terms

Key Idea: When related quantities change by the same factor, we call that relationship a proportional relationship.

1. PROPORTIONALITY—A QUICK RECAP

A proportional relationship between two quantities can be represented using ratios.

Example:

Idli batter: 2 cups rice : 1 cup urad dal $\rightarrow$ 2 : 1

Viswanath mixes 6 cups rice and 3 cups urad dal $\rightarrow$ 6 : 3

Puneet mixes 4 cups rice and 2 cups urad dal $\rightarrow$ 4 : 2

Cross-Multiplication Method

$$\begin{array}{ccc} 6 : 3 & & 4 : 2 \\ & \swarrow \quad \searrow & \\ 6 \times 2 = 12 & \text{and} & 3 \times 4 = 12 \end{array}$$

The products are equal (12). So, the ratios are proportional. (Their idlis will taste the same if all other ingredients are proportional too!)

General Rule (Two Ratios)

Two ratios $a : b$ and $c : d$ are proportional if

$$a \times d = b \times c \quad \text{or} \quad \frac{a}{c} = \frac{b}{d}$$

2. RATIOS IN MAPS

Many maps show a ratio called **Representative Fraction (RF)** (usually in the lower right corner).

RF = 1 : N means 1 unit on the map = N units on the ground (real geographical distance, not road distance).

Example:

$$\text{RF} = 1 : 60,00,000$$

1 cm on map
= 60,00,000 cm on ground
= 60 km



(Map not to scale)

How to find distance between two cities:

1. Measure the distance between the cities on the map using a ruler.
2. Multiply by the scale (N) to get the actual geographical distance.

Try This!

- Find the distance between Bengaluru and Chennai.
- Find the distance between Mangaluru and Chennai.
- Use different maps (with different scales) and check if you get approximately the same geographical distance.

Note to the Teacher:

Bring maps and atlases to class. Ask students to observe the scale (ratio) on maps. Let them find geographical distances between places of local interest and compare results with peers.

Map Making Activity:

Ask students to draw a sketch of their classroom using a scale (e.g., 1 : 50). Mark positions of objects (desks, board, fans, lights, etc.) using suitable symbols.

3. RATIOS WITH MORE THAN 2 TERMS

Ratios can have more than two terms if the quantities change by the same factor.

Example:

Viswanath's spice mix powder:

8 spoons coriander : 4 red chillies : 2 spoons toor dal : 1 spoon fenugreek (methi) seeds

Ratio = 8 : 4 : 2 : 1 (4 terms)

? Puneet has only 2 red chillies (half of Viswanath's 4 chillies). How much of the other ingredients should he use to make the same spice mix powder?

Since he has half the chillies, he must also take half of all other ingredients.

Puneet's mix: 4 : 2 : 1 : 0.5

(4 spoons coriander, 2 red chillies, 1 spoon toor dal, half spoon fenugreek seeds)

Both ratios are proportional:

$$8 : 4 : 2 : 1 :: 4 : 2 : 1 : 0.5$$

General Rule (Multiple Terms)

If two ratios with the same number of terms are proportional,

$$a : b : c : d :: p : q : r : s$$

then, $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s}$

Example 1: Mixing Paint

Ratio (Red : Blue : White) = 2 : 3 : 5

Yasmin has 10 litres of white paint. How much red and blue paint should she add?

- White corresponds to 5 parts.
- If 5 parts = 10 L $\rightarrow$ 1 part = $10 \div 5 = 2$ L
- Red = 2 parts = $2 \times 2 = 4$ L
- Blue = 3 parts = $3 \times 2 = 6$ L
- Total volume of purple paint = $4 + 6 + 10 = 20$ litres



Example 2: Cement Concrete Mix

Ratio (Cement : Sand : Aggregate) = 1 : 2 : 4

If 7 kg of cement is used, how much sand and aggregate are needed?

- Cement corresponds to 1 part.
- 1 part = 7 kg
- Sand = 2 parts = $2 \times 7 = 14$ kg
- Aggregate = 4 parts = $4 \times 7 = 28$ kg
- Total mix = $7 + 14 + 28 = 49$ kg



KEY DEFINITIONS

- **Ratio:** A way to compare two or more quantities using division ($a : b$ means "a to b").
- **Proportional relationship:** When related quantities change by the same factor.
- **Representative Fraction (RF):** The ratio of a distance on the map to the actual geographical distance on the ground.

IMPORTANT FORMULAS

- Two ratios $a : b$ and $c : d$ are proportional if $\frac{a}{c} \times d = b$ or $\frac{a}{c} = \frac{b}{d}$
- For multiple terms $a : b : c : d :: p : q : r : s \Rightarrow \frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s}$

KEY TAKEAWAYS

- ✓ Proportionality helps us compare quantities easily.
- ✓ Maps use a ratio (RF) to represent real distances.
- ✓ Ratios can have more than two terms.
- ✓ Keep the same ratio (or scale) to maintain the same relationship.

PROPORTIONAL REASONING -2

WORKSHEET-1

Section A: MCQs (Part-1) ($10 \times 1 = 10$ marks)

- If 4 notebooks cost ₹120, then the cost of 1 notebook is:
a) ₹20 b) ₹25 c) ₹30 d) ₹35
- Which of the following represents a proportional relationship?
a) $2:4 = 3:6$ b) $2:5 = 4:8$ c) $3:7 = 6:10$ d) $5:8 = 10:12$
- If 5 kg rice costs ₹250, then 8 kg rice costs:
a) ₹350 b) ₹400 c) ₹450 d) ₹500
- The ratio 15 : 20 in simplest form is:
a) 3 : 4 b) 4 : 5 c) 5 : 6 d) 15 : 20
- If x and y are proportional, then:
a) $x + y$ is constant b) $x - y$ is constant
c) x/y is constant d) xy is constant
- A car travels 120 km in 3 hours. Distance travelled in 5 hours at the same speed is:
a) 180 km b) 200 km c) 220 km d) 240 km
- If 6 pens cost ₹90, then 10 pens cost:
a) ₹120 b) ₹130 c) ₹150 d) ₹180
- Which pair is proportional?
a) 2, 4, 6, 8 b) 3, 5, 9, 15 c) 4, 6, 8, 12 d) 5, 7, 10, 15
- If 12 workers complete a job in 8 days, then the number of worker-days is:
a) 20 b) 96 c) 80 d) 100
- In a map scale of 1 cm = 5 km, 4 cm represents:
a) 10 km b) 15 km c) 20 km d) 25 km

MCQs (Part-2) ($10 \times 1 = 10$ marks)

11. A recipe requires flour : sugar = 5 : 2. If 15 cups of flour are used, how many cups of sugar are needed?

- a) 4 cups b) 5 cups c) 6 cups d) 8 cups

12. A map has a scale of 1 : 2,50,000. If the distance between two towns on the map is 8 cm, what is the actual distance?

- a) 10 km b) 15 km c) 20 km d) 25 km

13. Three friends share a prize money of ₹3600 in the ratio 2 : 3 : 4. The largest share is

- a) ₹800 b) ₹1000 c) ₹1200 d) ₹1600

14. A pie chart represents the favourite fruits of students. If 90° of the chart represents apples, what fraction of the students prefer apples?

- a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

15. A car travels 240 km in 4 hours. If the speed remains constant, how far will it travel in 7 hours?

- a) 360 km b) 400 km c) 420 km d) 480 km

16. If 12 workers complete a job in 15 days, how many days will 20 workers take to complete the same job?

- a) 6 days b) 8 days c) 9 days d) 10 days

17. A painter uses Red : Blue : Yellow = 4 : 3 : 1 to prepare a colour mixture. If Yellow = 5 litres, how much Blue paint is required?

- a) 10 litres b) 12 litres c) 15 litres d) 20 litres

18. A school bus travelling at 50 km/h reaches a destination in 4 hours. If the speed increases to 80 km/h, the journey will take

- a) 2.5 hours b) 3 hours c) 3.5 hours d) 4 hours

19. In a pie chart, the sector representing Science measures 126° . If there are 240 students, how many students prefer Science?

- a) 72 b) 80 c) 84 d) 90

20. Which of the following pairs of quantities are inversely proportional?

- a) Distance travelled and time taken at constant speed
b) Number of notebooks bought and total cost (same price per notebook)
c) Number of workers and time taken to complete the same work
d) Age of a person and height of a tree

WORKSHEET-2

Section B: Short Answer ($10 \times 2 = 20$ marks)

1. Find the unit rate if 7 kg apples cost ₹280.

2. Are the ratios 12:18 and 8:12 equivalent? Give reason.

3. A train travels 180 km in 3 hours. Find its speed.

4. Simplify the ratio 24 : 36.

5. If 3 m cloth costs ₹240, find the cost of 5 m cloth.

6. Complete the proportion: $6 : 9 = \underline{\hspace{1cm}} : 15$

7. A shop sells 8 bottles of juice for ₹320. What is the cost of one bottle?

8. Write one example from daily life where proportional reasoning is used.

9. Find the missing term: $4 : 7 = 12 : \underline{\hspace{1cm}}$

10. If 15 notebooks cost ₹450, what will be the cost of 20 notebooks?

Section C: Short Answer ($6 \times 3 = 18$ marks)

11. A cyclist covers 45 km in 3 hours. How far will he travel in 7 hours at the same speed?

12. A recipe needs 250 g flour for 5 persons. How much flour is needed for 8 persons?

13. The cost of 12 chairs is ₹5400. Find the cost of 18 chairs.

14. A map uses a scale of $1 \text{ cm} = 8 \text{ km}$. What actual distance does 6.5 cm represent?

15. A water tanker fills 600 litres in 10 minutes . How much water will it fill in 18 minutes ?

16. Verify whether the following are proportional: $18 : 24$ and $27 : 36$

Section D: Long Answer ($4 \times 5 = 20$ marks)

17. A school bus travels 150 km using 15 litres of diesel. How much diesel will be required for 250 km ?

18. The wages of 8 workers for one day are $\text{₹}3200$. Find the wages of 15 workers for one day.

19. A grocer sells 18 kg sugar for ₹900. Find the cost of 1 kg sugar. Find the cost of 25 kg sugar.

20. A factory produces 450 bottles in 5 hours. How many bottles are produced in 8 hours? How many bottles are produced in 12 hours?

SECTION E: Case Study Based Questions ($4 \times 2 = 8$ marks)

21. A map of a state uses the scale 1 cm = 20 km. The distance between City A and City B is shown as 7.5 cm on the map.

Based on the above information, answer the following questions:

- a) What is the actual distance between City A and City B?
- b) What distance on the map would represent an actual distance of 240 km?
- c) If a car travels from City A to City B at an average speed of 50 km/h, how much time will it take to reach City B?
- d) If another city is 300 km away from City A, how many centimetres apart should the two cities be shown on the map?

22. A trekking map uses the scale $1 \text{ cm} = 8 \text{ km}$. The distance between the Base Camp and Hill Point is shown as 9 cm on the map. Based on the information, answer the following questions:

- a) What is the actual distance between the Base Camp and Hill Point?
- b) If trekkers walk at an average speed of 4 km/h, how much time will they take to reach Hill Point?
- c) What map distance will represent an actual distance of 120 km?
- d) The distance between Hill Point and River Point is shown as 5.5 cm on the map. Find the actual distance between these two places.

Self Assessment:

I understood all the concepts: ☐ Yes ☐ No

I need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Parent's Assessment:

My ward understood all the concepts: ☐ Yes ☐ No

He/She need more practice: ☐ Yes ☐ No

Name of Concepts having doubts/Not Cleared: _____

Signature of Parent:

Teacher's Assessment

Answers of Worksheet-1

Q.No.	1	2	3	4	5	6	7	8	9	10
Ans.	b	a	b	a	c	b	c	c	b	c
Q.No.	11	12	13	14	15	16	17	18	19	20
Ans.	c	c	d	a	c	c	c	a	c	c

Answers of Worksheet-2

Q.No.	1	2	3	4	5	6
	₹40/kg	Yes	60km/hr	2:3	₹400	10
	7	8	9	10	11	12
Ans.	₹40	Any suitable example	21	₹600	105	400
	13	14	15	16	17	18
	₹8100	52m	1080	yes	25	₹6000
	19	20				
	₹1250	720 bottles, 1080 bottles	a) 150 km b) 12 cm c) 3 hours d) 15 cm		a) 72 km b) 18 hours c) 15 cm d) 44 km	

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EXPLORING SOME GEOMETRIC THEMES

In This Chapter

- ✓ Fractals (self-similar shapes)
- ✓ Visualising Solids in Different Ways



We explore two geometric themes in this chapter:

1. **Fractals** – self-similar shapes that repeat the same pattern at smaller and smaller scales.
2. **Visualising Solids** – understanding the same solid from different views and cross-sections.

1. FRACTALS

Key Idea

A fractal is a shape that shows **self-similarity** – the same or similar pattern appears again and again at smaller and smaller scales.

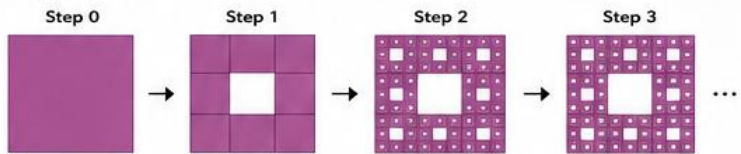
Examples in Nature

Ferns, trees, clouds, coastlines, mountains, lightning and many more.



A. SIERPINSKI CARPET

Start with a square. Divide it into 9 equal squares and remove the central square. Repeat the same process on the remaining 8 squares, and so on.



Number of Squares that Remain (R_n)

Each remaining square produces 8 squares in the next step.

$$R_{n+1} = 8 R_n$$

$$R_0 = 1 \Rightarrow R_n = 8^n$$

Number of Holes (H_n)

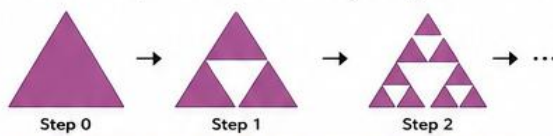
Each remaining square creates a new hole and all previous holes remain.

$$H_{n+1} = H_n + R_n$$

$$H_0 = 0, H_1 = 1, H_2 = 1 + 8, H_3 = 1 + 8 + 8^2, \dots$$

B. SIERPINSKI TRIANGLE (GASKET)

Start with an equilateral triangle. Join midpoints of its sides to form 4 smaller congruent triangles and remove the central one. Repeat on the 3 remaining triangles.



★ Joining the midpoints of an equilateral triangle divides it into 4 identical equilateral triangles. (Hint: Corner triangles are isosceles.)

Figure it Out

1. Draw the first few steps (at least till Step 2).
2. Find the number of holes and triangles that remain at each step.
3. Find the area of the region remaining at the n^{th} step. (Take the starting triangle to have area = 1 sq. unit.)

C. KOCH SNOWFLAKE

Start with an equilateral triangle. Each side is divided into 3 equal parts. On the middle part, an outward equilateral triangle is built and the middle part is removed. Repeat on every side of the new shape, and so on.

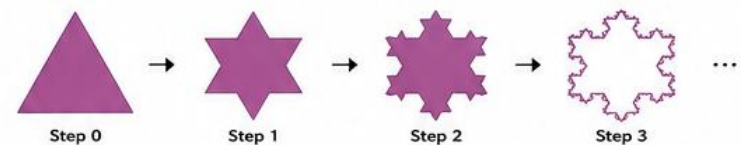


Figure it Out

1. Draw the first few steps (at least till Step 2).
2. Find the number of sides at the n^{th} step.
3. Find the perimeter at the n^{th} step. (Take the starting triangle to have side length = 1 unit.)

FRACTALS IN ART

Fractal patterns have been used in art and architecture for centuries!

- ◆ **Indian temples:** Structures built of smaller copies of the whole form.
- ◆ **African art:** Traditional patterns on textiles (e.g., Nigerian Fulani wedding blankets).
- ◆ **Islamic art:** Intricate repeating fractal-like geometric motifs.



Kandariya Mahadev Temple, Khajuraho, Madhya Pradesh (c. 1025 CE)



Nigerian Fulani Wedding Blanket



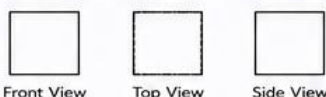
Islamic Geometric Motif

2. VISUALISING SOLIDS IN DIFFERENT WAYS

We can look at the same solid in different ways:

A. DIFFERENT VIEWS

A solid looks different from different directions.



B. CROSS-SECTIONS

A slice (cut) of a solid gives a 2D shape.



C. EXPLORE & SKETCH

- ◆ Think, imagine, cut and draw different views and cross-sections to understand solids better.



KEY DEFINITIONS

- **Fractal:** A shape that shows self-similarity at different scales.
- **Self-similarity:** Similar pattern repeats at smaller and smaller sizes.
- **Square Hole / Triangle Hole:** A part removed at each step in constructing a fractal.
- **Cross-section:** The 2D shape obtained by slicing a solid with a plane.

IMPORTANT FORMULAS

Sierpinski Carpet

$$R_{n+1} = 8 R_n \Rightarrow R_n = 8^n$$

$$H_{n+1} = H_n + R_n \quad (\text{with } H_0 = 0)$$

Sierpinski Triangle

$$R_{n+1} = 3 R_n \Rightarrow R_n = 3^n$$

$$H_{n+1} = H_n + R_n \quad (\text{with } H_0 = 0)$$

Koch Snowflake (perimeter)

$$P_{n+1} = \frac{4}{3} P_n \Rightarrow P_n = \left(\frac{4}{3}\right)^n P_0$$

KEY TAKEAWAYS

- ✓ Fractals are everywhere – in nature, art and mathematics.
- ✓ They are built using simple rules repeated many times.
- ✓ Counting patterns (squares, triangles, holes, sides) helps us understand fractals.
- ✓ Visualising solids from different views and cross-sections builds spatial understanding.